Lecture 12: Laser Doppler Velocimetry (LDV) & Global Doppler Velocimetry (GDV) - Part 01

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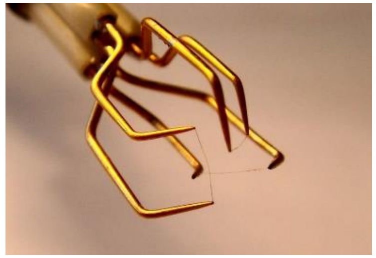
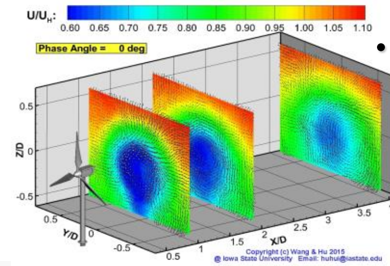
❑ Various measurement Techniques for Thermo-Flow Studies

Velocity, temperature, pressure, density (concentration), etc..

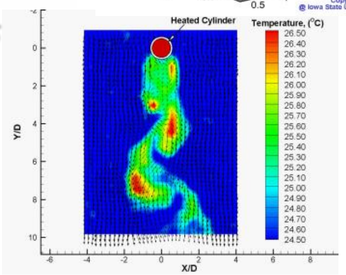


• *Hotwire probe*

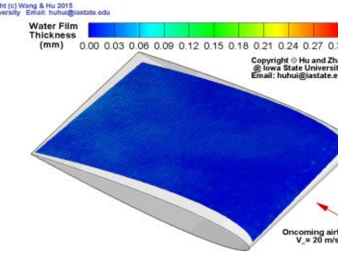
• *Thermocouples*



• *Stereoscopic PIV*



• *MTV & MTT*



• *DIP measurements*

Thermal-Fluids measurement techniques

- **Intrusive techniques**
  - Pitot probe
  - hotwire, hot film
  - thermocouples
  - etc...
- **Non-intrusive techniques**
  - **particle-based techniques**
    - Laser Doppler Velocimetry (LDV)
    - Planar Doppler Velocimetry (PDV)
    - Particle Image Velocimetry (PIV)
    - etc...
  - **molecule-based techniques**
    - Laser Induced Fluorescence (LIF)
    - Molecular Tagging Velocimetry (MTV)
    - Molecular Tagging Therometry (MTT)
    - Digital Image Projection (DIP)
    - Pressure Sensitive Paint (PSP)
    - Temperature Sensitive Paint (TSP)
    - Quantum Dot Imaging
    - etc ...

Flow velocity V_f = Velocity of particles or molecule Tracers, V_{tracer}

❑ Particle-based Flow Diagnostic Techniques

- *Seeded the flow with small particles (~ µm in size)*
- *Assumption: the particle tracers move with the same velocity as local flow velocity!*

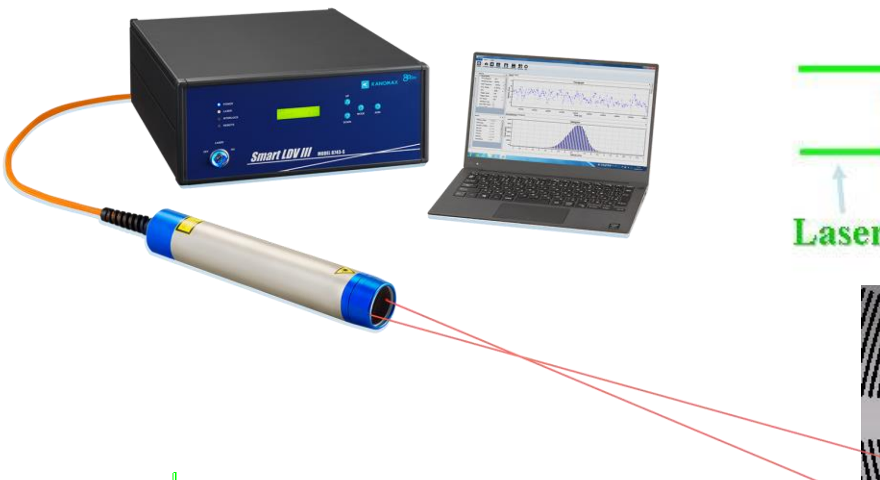
Flow velocity V_f = **Particle velocity V_p**

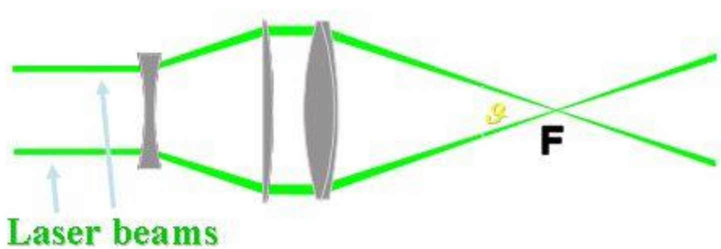
Measurement of particle velocity

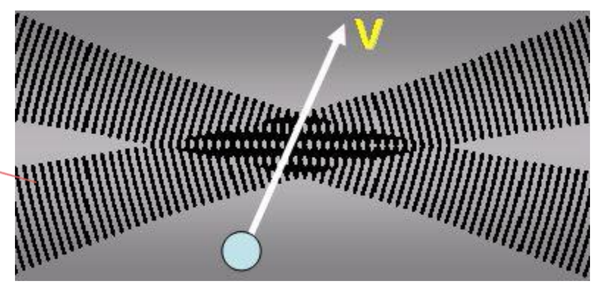
- *Smoke visualization*

❑ Laser Doppler Velocimetry (LDV)

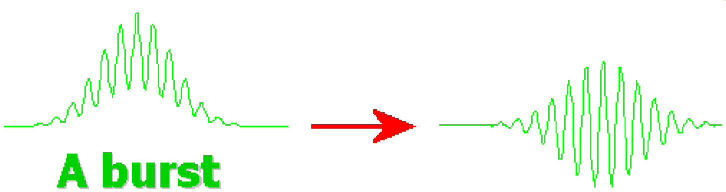
- *Laser Doppler velocimetry (LDV, also known as laser Doppler anemometry, or LDA) is a technique for measuring the direction and speed of fluids like air and water.*
- *In its simplest form, LDV crosses two beams of collimated, monochromatic laser light in the flow of the fluid being measured.*
- *A microscopic pattern of bright and dark stripes forms in the intersection volume.*
- *Small particles in the flow pass through this pattern and reflect light towards a detector, with a characteristic frequency indicating, via the Doppler effect, the velocity of the particle passing through the probe volume.*







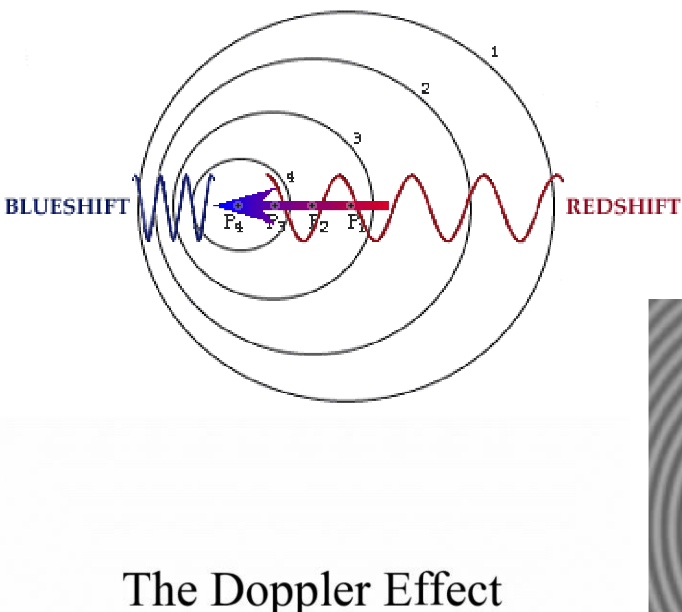
$$v_{\perp} = \frac{\lambda}{2\sin\frac{\vartheta}{2}} f$$

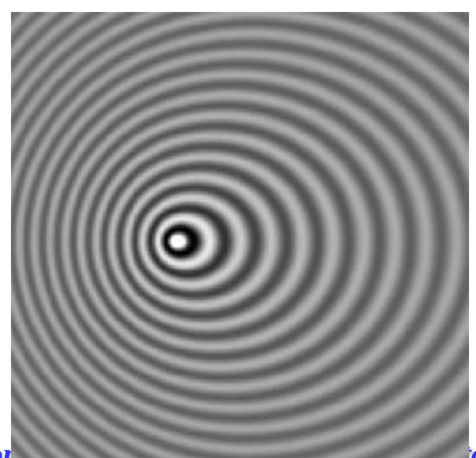


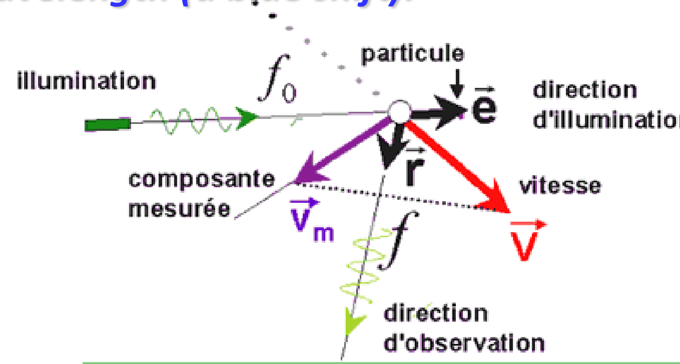
- *Interference fringes*

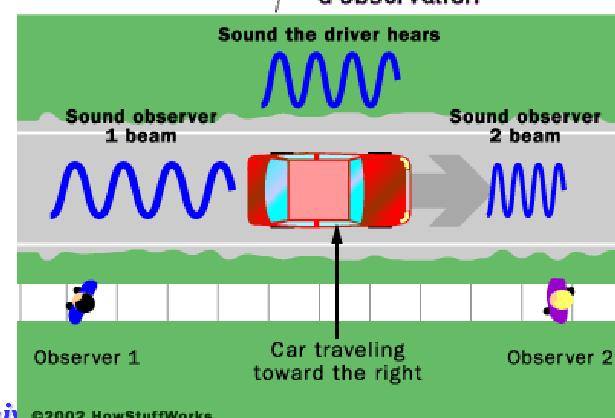
❑ Doppler Shift

- *The Doppler effect, named after* **Christian Doppler (an Austrian mathematician and physicist)***, is the change in frequency and wavelength of a wave that is perceived by an observer moving relative to the source of the waves.*
- *Light from moving objects will appear to have different wavelengths depending on the relative motion of the source and the observer.*
- *Observers looking at an object that is moving away from them see light that has a longer wavelength than it had when it was emitted (a red shift), while observers looking at an approaching source see light that is shifted to shorter wavelength (a blue shift).*







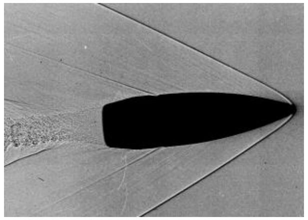


❑ Doppler Shift

a. Stationary Sound Source

b. Source moving with Vsource < Vsound

c. Source moving with Vsource = Vsound (Mach 1 - breaking the sound barrier)



d. Source moving with Vsource > Vsound (Mach 1.4 - supersonic)

❑ Fundamentals of LDV

- *For waves that travel through a medium (sound, ultrasound, etc...) the relationship between observed frequency f' and emitted frequency f is given by:*

$$f' = \left(\frac{v}{v \pm v_s}\right) f$$

  - *where*
    - *v is the speed of waves in the medium*
    - *v_s is the velocity of the source*
- *For waves that travel at the speed of light, such as laser light, the relationship between observed frequency f' and emitted frequency is given by:*

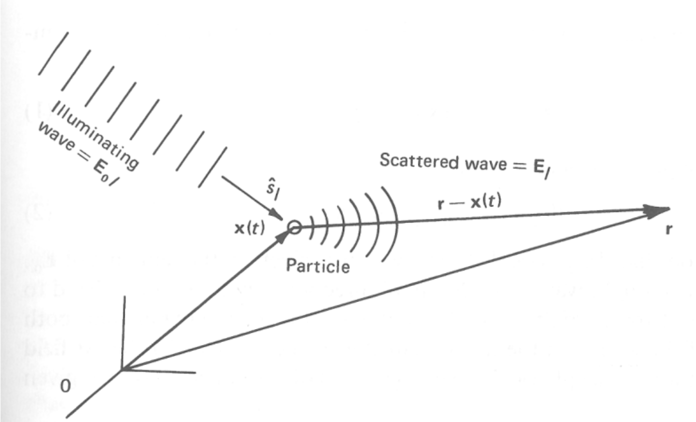


Figure 4.1 Light scattered by a moving particle

$$f = \left(1 - \frac{v_{s,r}}{c}\right) f_0$$

$$\Delta f = -\frac{v_{s,r}}{c} f_0 = -\frac{v_{s,r}}{\lambda_0}$$

- *Because the detected frequency increases for objects moving toward the observer, the object's velocity must be subtracted when motion is moving toward the observer. (This is because the source's velocity is in the denominator.)*
- *Conversely, detected frequency decreases when the object moves away, and so the object's velocity is added when the motion is away.*

❑ Fundamentals of LDV

- *Take the coordinate system to be at rest with respect to the medium, whose speed of light wave is c. There is a source s moving with velocity V_s and emitting light waves with a frequency f_s.*
- *There is a detector r moving with velocity V_r, and the unit vector from s to r is n i.e.* $\mathbf{r}_r - \mathbf{r}_s = \mathbf{n}|\mathbf{r}_r - \mathbf{r}_s|$
- *Then the frequency f_r at the detector is found from*

$$\frac{f_r}{f_s} = \frac{1 - \mathbf{n}\cdot\mathbf{v}_r/c}{1 - \mathbf{n}\cdot\mathbf{v}_s/c}$$

- *If c>>V_s, then the change in frequency depends mostly on the relative velocity of the source and detector.*

$$\frac{f_r}{f_s} \approx 1 - \mathbf{n}\cdot(\mathbf{v}_r - \mathbf{v}_s)/c$$

$$\frac{\Delta f}{f_s} = \frac{f_r - f_s}{f_s} = -\hat{n}\cdot\frac{\vec{V}_r - \vec{V}_s}{c}$$

$$\left.\begin{array}{l}\hat{n} = \hat{e}_r - \hat{e}_i \\ V_r = 0\end{array}\right\} \Rightarrow \frac{\Delta f}{f_s} = \frac{\vec{V}_s\cdot(\hat{e}_r - \hat{e}_i)}{c} = \frac{V_\phi \cdot 2\sin(\frac{\phi}{2})}{c}$$

$$\Rightarrow \Delta f = \frac{V_\phi \cdot 2\sin(\frac{\phi}{2}) f}{f\lambda} = \frac{V_\phi \cdot 2\sin(\frac{\phi}{2})}{\lambda}$$

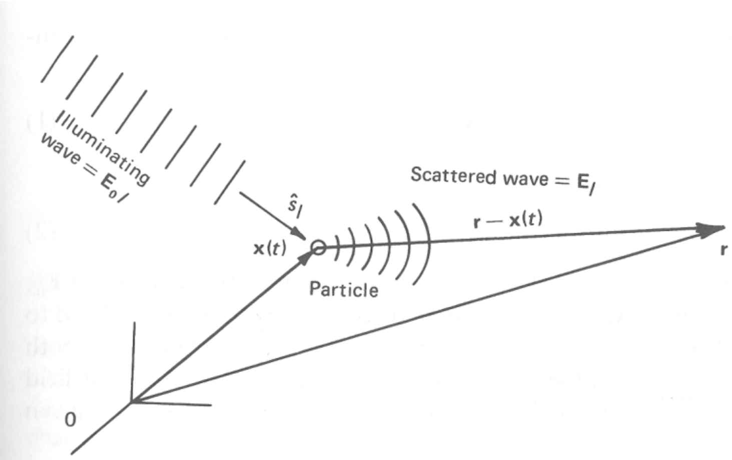


Figure 4.1 Light scattered by a moving particle

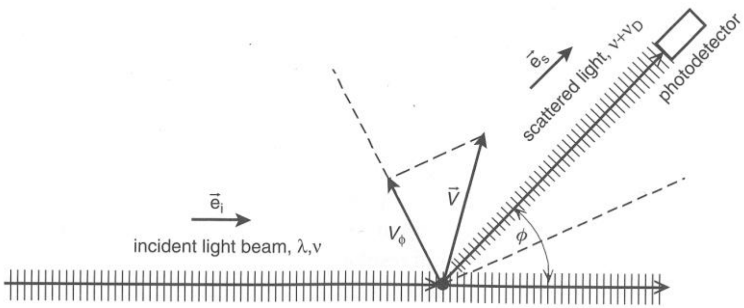


Figure 11.9. Sketch of a laser beam light scattered by a moving particle.

❑ Fundamentals of LDV

- *By using a laser bean of wavelength λ=488nm (Argon-Ion laser), the maximum Doppler shift from a particle moving with a velocity of V would be:*
  - *V=1.0m/s $\Delta f \approx$ 4.1 MHz*
  - *V=10.0m/s $\Delta f \approx$ 41 MHz*
  - *V=100.0m/s $\Delta f \approx$ 410 MHz*
  - *V=1000m/s $\Delta f \approx$ 4100 MHz*
- *However, since C = 2.998×10^8 m/s, λ=488nm, then, f=c/ λ= 1.4 ×10 ^{9} MHz. the Doppler shift in frequency is very small compared with the frequency of the source laser light.*
- *In practice, it is always quit difficult to measure the Doppler shift of frequency accurately for low-speed flows by measuring the received total frequency directly.*
- *Dual-beam LDV technique was developed to measure the relative frequency change due to the Doppler shift other than the total frequency.*

$$\Delta f = \frac{V_\phi \cdot 2\sin(\frac{\phi}{2})\, f}{f\lambda} = \frac{V_\phi \cdot 2\sin(\frac{\phi}{2})}{\lambda}$$

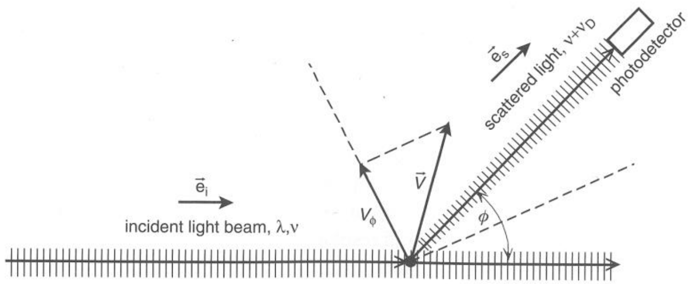


Figure 11.9. Sketch of a laser beam light scattered by a moving particle.

https://www.youtube.com/watch?v=JxfWi85l0hg

❑ Fundamentals of Dual-Beam LDV

If the intensity of each scattered beam collected by the photo detector varies sinusoidal ,

$$A_i \sin 2\pi (f + \Delta f_i)t, \quad i = 1, 2.$$

Then, the optical mixing of these beams on the photoditec tor (heterodyn ing process) produces an output vol tage E that is proportion al to the squire of the combined light intensity.

$$E \sim \{A_1 \sin 2\pi (f + \Delta f_1)t + A_2 \sin 2\pi (f + \Delta f_2)t\}$$
$$= A_1^2 \sin^2 2\pi (f + \Delta f_1)t + A_2^2 \sin^2 2\pi (f + \Delta f_2)t + 2A_1A_2[\sin 2\pi (f + \Delta f_1)t][\sin 2\pi (f + \Delta f_1)t]$$
$$= A_1^2 \sin^2 2\pi (f + \Delta f_1)t + A_2^2 \sin^2 2\pi (f + \Delta f_2)t + A_1A_2[\cos 2\pi (\Delta f_1 - \Delta f_2)t] - \cos 2\pi (2f + \Delta f_1 + \Delta f_2)t]$$
$$= \underbrace{A_1^2 \sin^2 2\pi (f + \Delta f_1)t + A_2^2 \sin^2 2\pi (f + \Delta f_2)t - \cos 2\pi (2f + \Delta f_1 + \Delta f_2)t]}_{high-frequency} + \underbrace{A_1A_2[\cos 2\pi (\Delta f_1 - \Delta f_2)t]}_{low-frequency}$$

$$If \ we \ define, \Delta f_1 - \Delta f_2 = f'$$

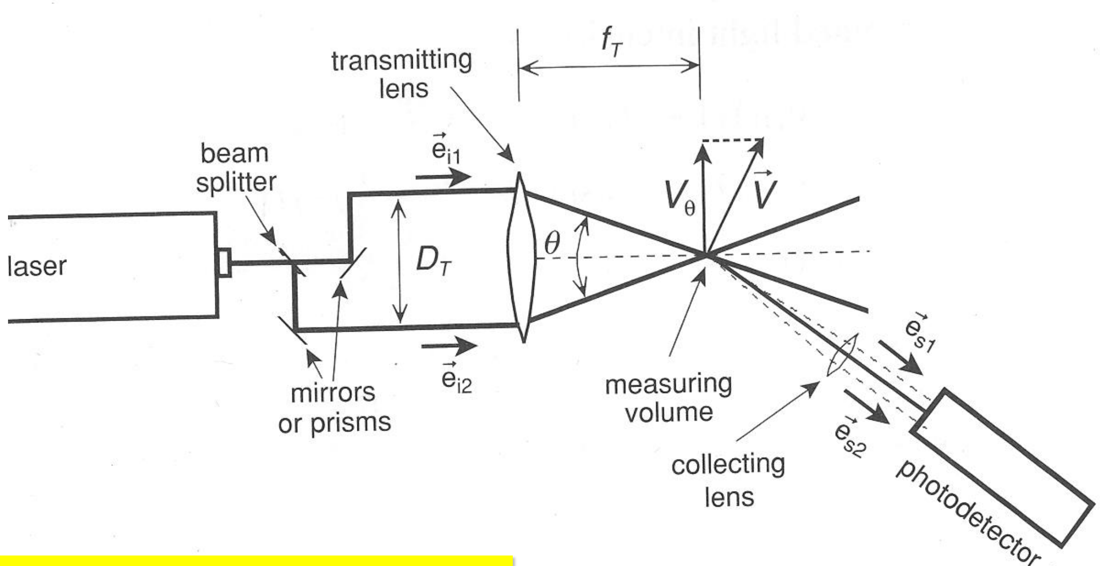
$$then: E \sim a + b \sin 2\pi f' t$$

$$f' = \Delta f_1 - \Delta f_2 = \frac{\vec{V} \cdot (\vec{e}_{s1} - \vec{e}_{i1})}{\lambda} - \frac{\vec{V} \cdot (\vec{e}_{s2} - \vec{e}_{i2})}{\lambda}$$

Since $\vec{e}_{i1} = \vec{e}_{i2}$,
then

$$f' = \frac{\vec{V} \cdot (\vec{e}_{i2} - \vec{e}_{i1})}{\lambda} = \frac{2\sin(\frac{\theta}{2})}{\lambda} V_\theta$$

$$\Rightarrow V_\theta = \frac{\lambda}{2\sin(\frac{\theta}{2})} f'$$



- ***The above equation is independent of observation angle!***

❑ Generated Fringes for the Dual-Beam LDV

Fring spacing : $\delta = \dfrac{\lambda}{2\sin(\theta/2)}$

Fring number : $N = \dfrac{4}{\pi}\dfrac{D_T}{d_e}$;

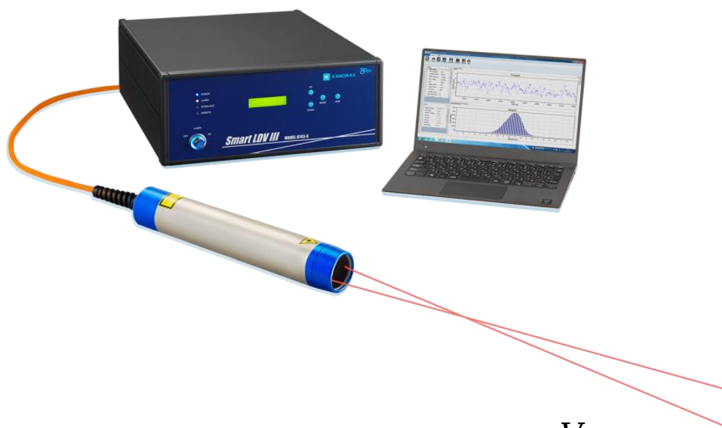
$D_T = 2 f_T \sin(\theta/2)$

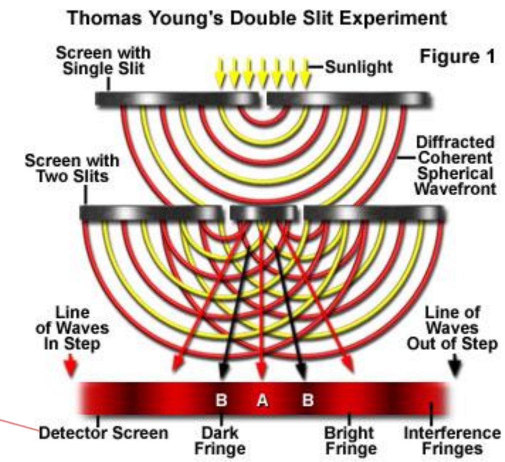
Frequency of the scattering light :

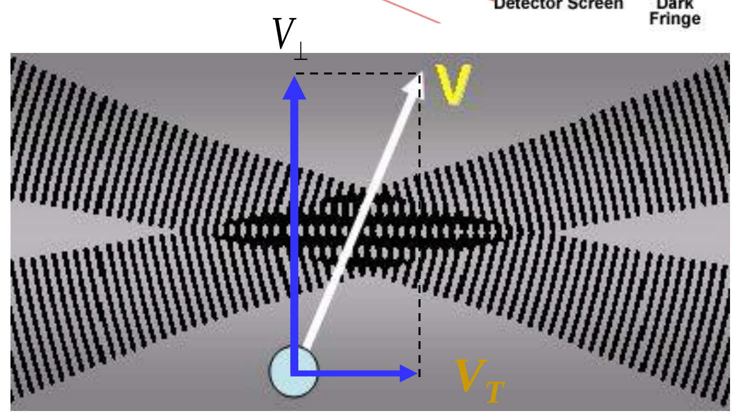
$$f = \frac{V_\perp}{\delta} = \frac{2\sin(\theta/2)}{\lambda} V_\perp$$

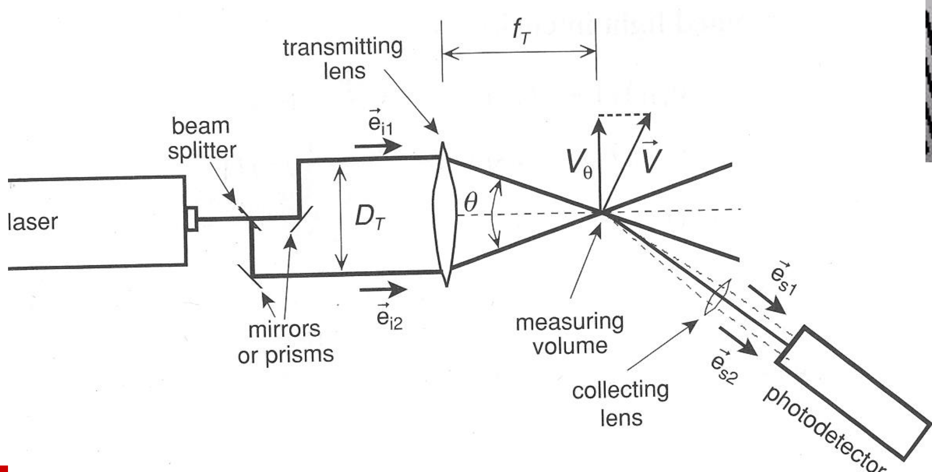
Frequency shift according to Doppler shift theo ry :

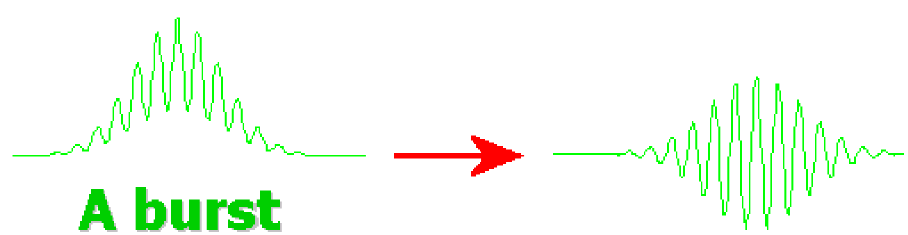
$$f = \frac{2\sin(\theta/2)}{\lambda} V_\perp$$











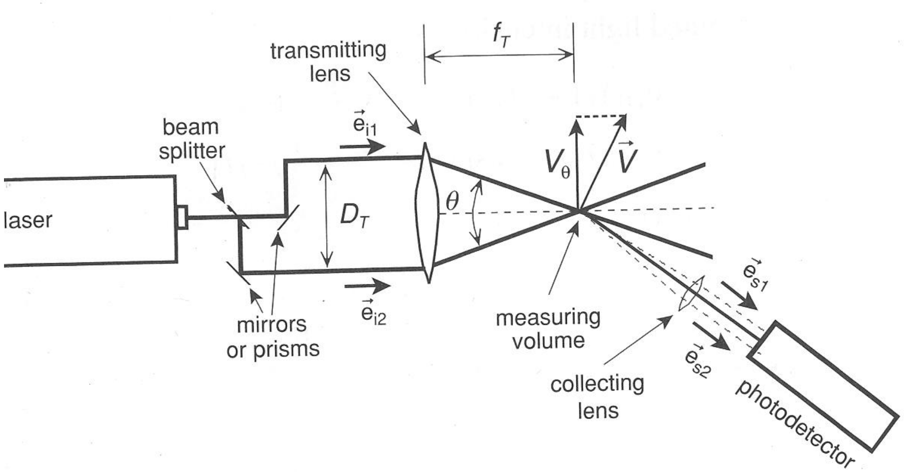
❑ Fundamentals of Dual-Beam LDV

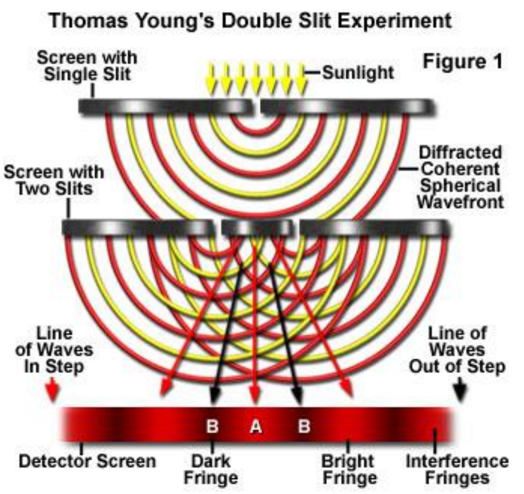
Width : $d_{fe} = \dfrac{4 f_T \lambda}{\pi\, d_e}$

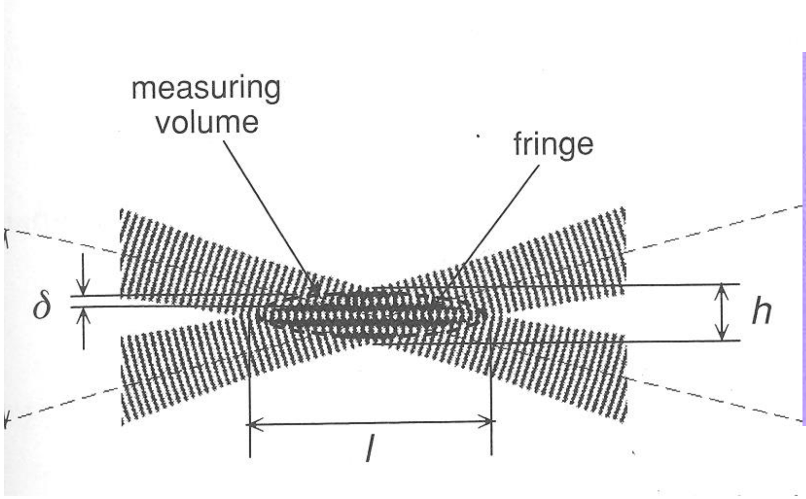
heigth : $h = \dfrac{d_{fe}}{\cos(\theta/2)}$

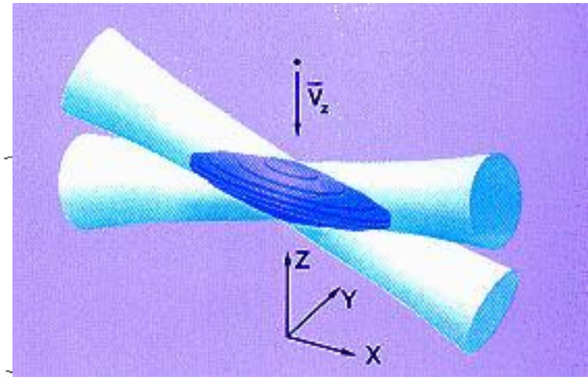
length : $l = \dfrac{d_{fe}}{\sin(\theta/2)}$

Volume : $Volume = \dfrac{\pi\, d^3{}_{fe}}{6\sin(\theta/2)\cdot\cos(\theta/2)}$









Beam Expander of a Dual-Beam LDV

- *A beam expander is a commonly used LDV accessory, whose function is to reduce the size of the measuring volume.*
- *This improves the spatial resolution of velocity measurement while also improving the amplitude resolution as a result of increased light power density within the measuring volume.*
- *A beam expander consists of a diverging lens and a converging lens, in addition to the transmitting lens.*
- *The beam expansion ratio: $E_x = d_{ex}/d_e = D_{Tx} / D_T$*

$$\theta_x = 2\sin^{-1}\left[\frac{E_x f_T}{f_{Tx}} \sin(\theta/2)\right]$$

f_{Tx} is the focal length of the transmitting lens

$$\text{if } f_{Tx} = f$$

$$\text{then}: d_{fex} = \frac{1}{E_x} d_{fe} \approx \frac{4 f_T \lambda}{\pi d_{ex}}$$

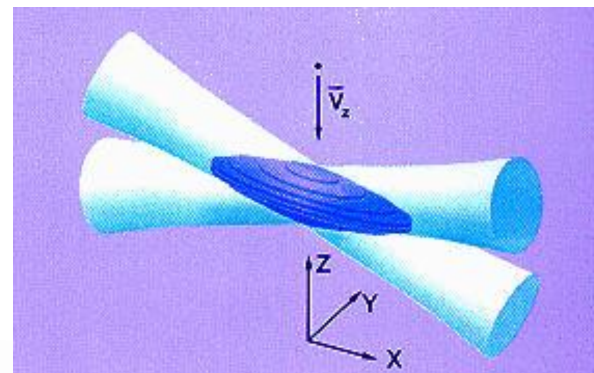
$$\text{Fringe Numbers}: \quad N = \frac{4}{\pi}\frac{D_T}{d_e} = \frac{4}{\pi}\frac{D_{Tx}}{d_{ex}};$$

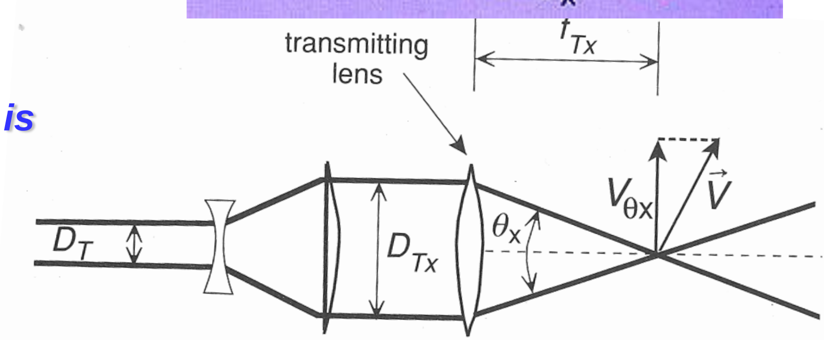
$$\text{Width}: \quad d_{fe} = \frac{4 f_T \lambda}{\pi d_e}$$

$$\text{heigth}: \quad h = \frac{d_{fe}}{\cos(\theta/2)}$$

$$\text{length}: \quad l = \frac{d_{fe}}{\sin(\theta/2)}$$

$$\text{Volume}: \quad Volume = \frac{\pi d^3_{fe}}{6\sin(\theta/2)\cdot\cos(\theta/2)}$$





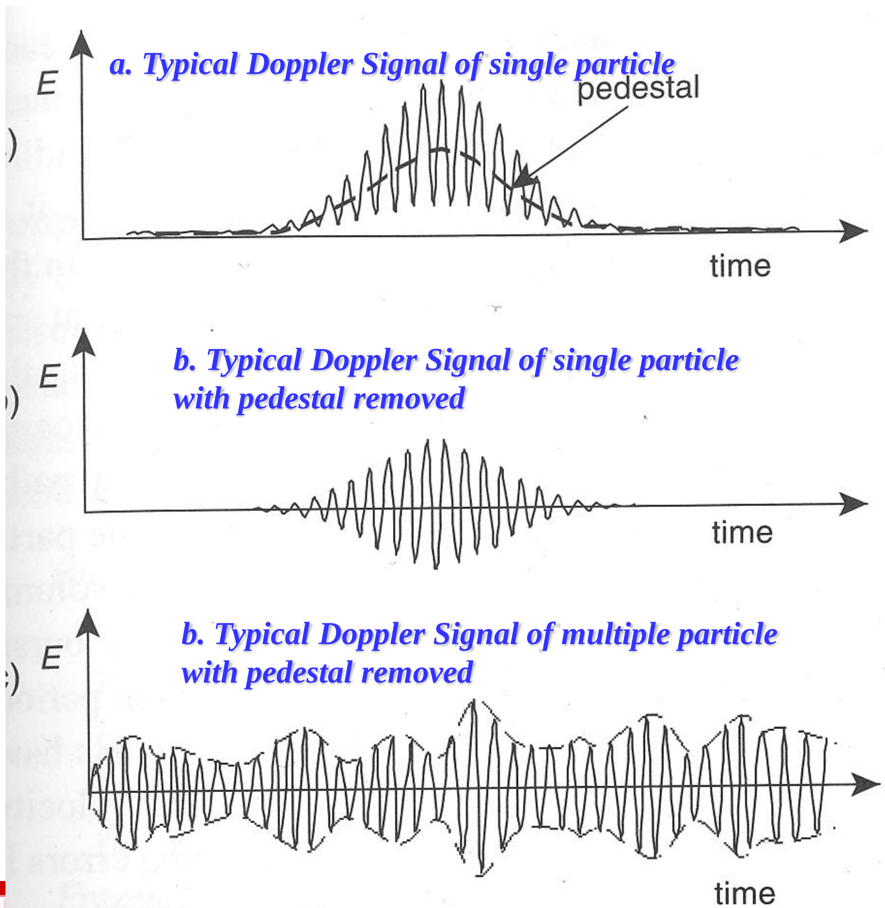
- *The fringe number within the measurement volume is remain unaffected.*
- *The fringe spacing will reduced by a factor of Ex,*
- *The measurement volume reduced by a factor ~Ex^4*
- *The SNR is also increased!*

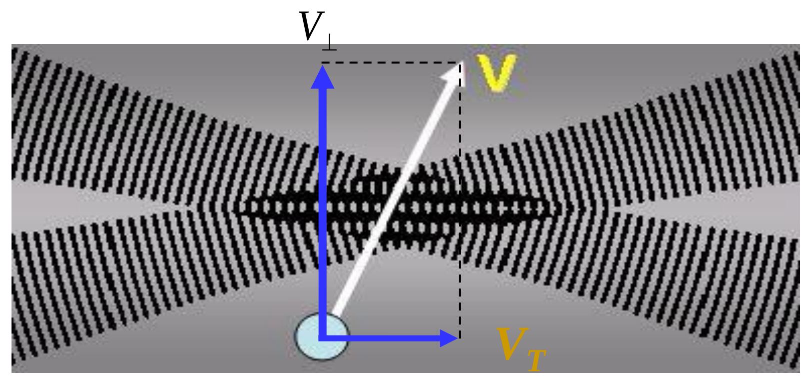
❑ Beam Expander of a Dual-Beam LDV

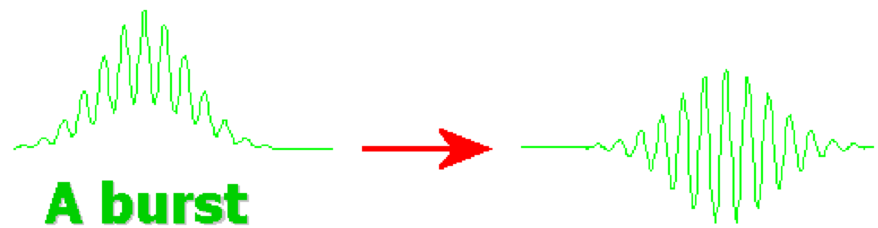
Frequency of the burst signal

$$f = \frac{V_\perp}{\delta} = \frac{2\sin(\theta/2)}{\lambda} V_\perp$$

$$V_\perp = \frac{f\lambda}{2\sin(\theta/2)}$$







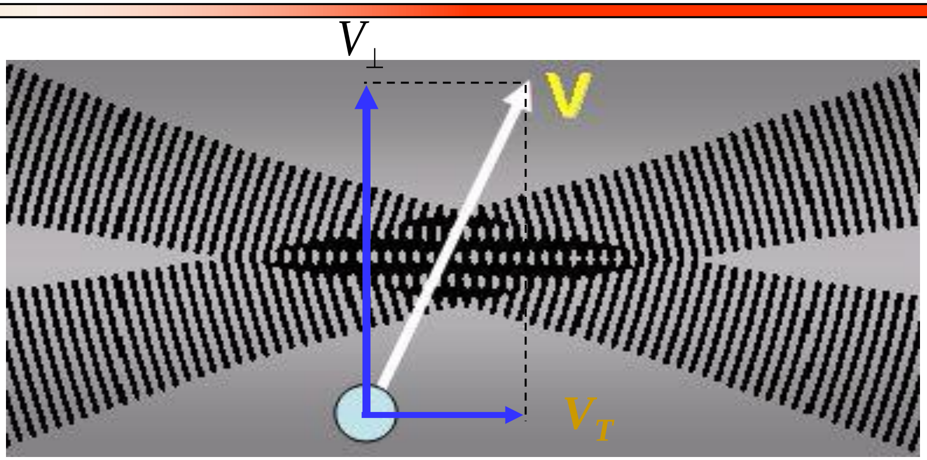
- *The pedestal is due to the intensity of the laser beam is usually has a Gauss Distribution.*
- *The scattering signal is depending on the size and reflective index of the particle and the its position in the measuring volume.*
- *Burst intensity: the number of the particles crossing the measuring volume*

❑ Beam Expander of a Dual-Beam LDV

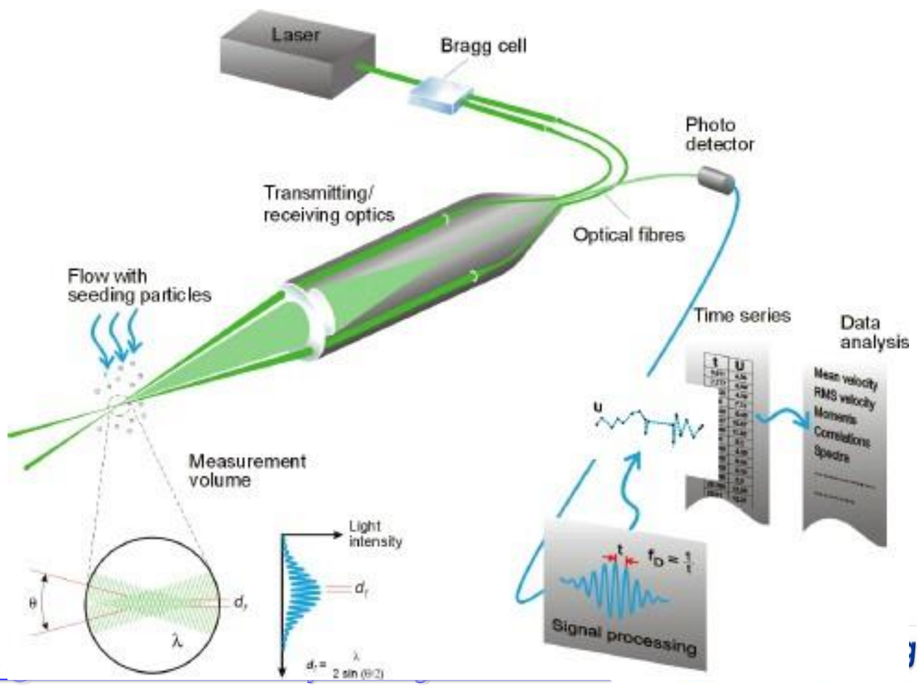
Frequency of the burst signal

$$f = \frac{V_\perp}{\delta} = \frac{2\sin(\theta/2)}{\lambda} V_\perp$$

$$V_\perp = \frac{f\lambda}{2\sin(\theta/2)}$$



- ***The relationship between the frequency shift and the velocity is equally valid for both senses of direction of velocity.***
- ***Bragg cell is used to remove this ambiguity.***
- ***The frequency shift obtained by the Bragg cell makes the fringe pattern move at a constant velocity.***
- ***Particles which are not moving will generate a signal of the shift frequency f_{shift}. The velocities Vp and V_{shift} will generate signal frequencies f_p and V_{shift} respectively.***



$$f = f_{particle} + f_{shift}$$

$$V_{measured} = V_p + V_{shift}$$

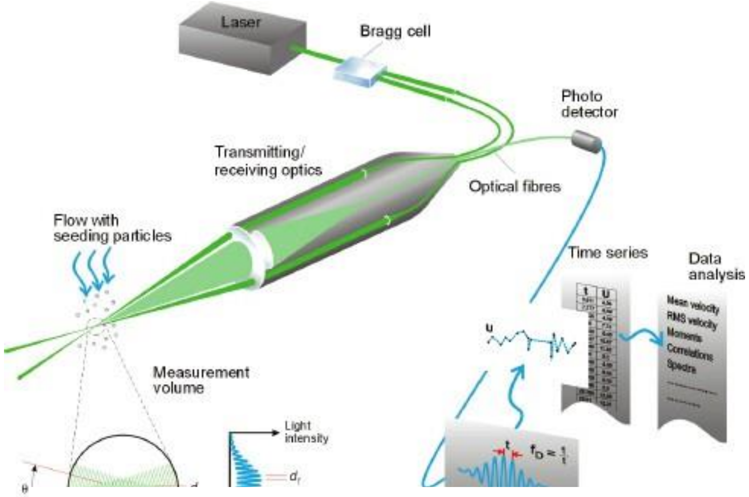
$$then:$$

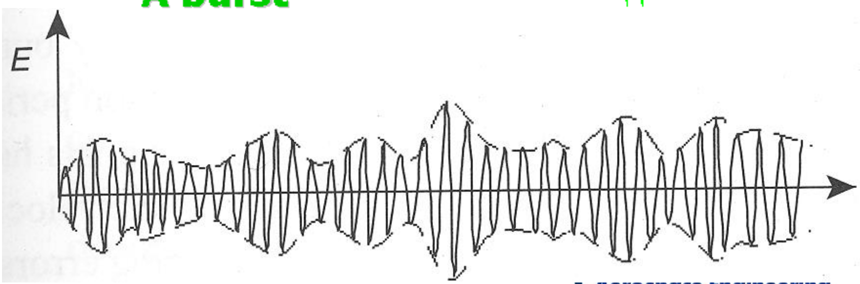
$$V_p = V_{measured} - V_{shift}$$

❑ Beam Expander of a Dual-Beam LDV

$$V_{\perp} = \frac{f\lambda}{2\sin(\theta/2)}$$

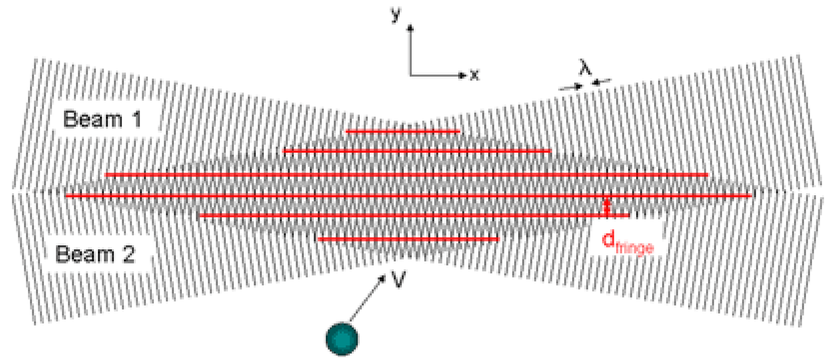
- *The signal is usually band pass filtered to remove the pedestal and high-frequency noise.*
- *A unit named as signal processor is used to determine the Doppler frequency.*
- *The signal processor includes:*
  - *Burst analyzer: FFT to get the power spectrum, then determine the Doppler frequency.*
  - *Frequency counters: count on the number of zero crossings of the filtered bursts. Determine the particle velocity as the ratio between the fringe space over the averaged time between the two zero crossing.*
  - *Frequency trackers: used for high particle density case. They contain an electric oscillator, which scans a frequency range and locks at the Doppler frequency, providing an analogue output proportional to it. Advantage is to provide analgue output, but has only limited dynamic range and need for heavy seeding.*
  - *Photo correlators: detect the emission of individual photons and correlated them with respect to their times of arrival to computer the time delay for peak correlation. Their advantage over other processor is that they can operate with very low light intensity and noisy signal. But their frequency range is limited, and quit time consuming.*





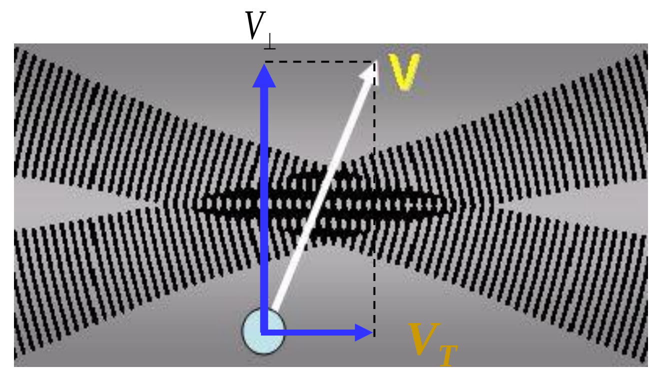
❑ Errors and uncertainty in LDV measurements

- *Fringe divergence uncertainty:*
  - *if the beams do not intersect at their waists, the fringes will not be parallel planes.*

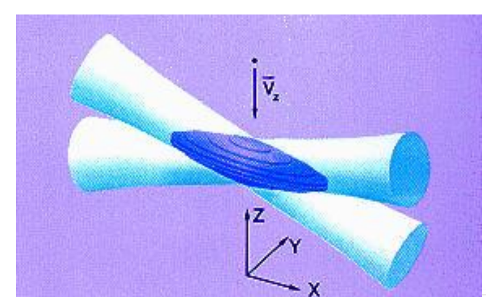


- *Velocity bias: due to many particles with different velocity passing the measuring volume.*

$$\frac{\overline{U}_m}{\overline{U}} \approx 1 + \frac{\overline{u^2}}{\overline{U}^2}$$



- *Directional bias: due to the small angle between the particle velocity and fringe direction.*



❑ Summary of LDV technique

https://www.youtube.com/watch?v=WL-7-47zWCY&t=7s