Lecture 15: Particle Image Velocimetry (PIV) - Part 02

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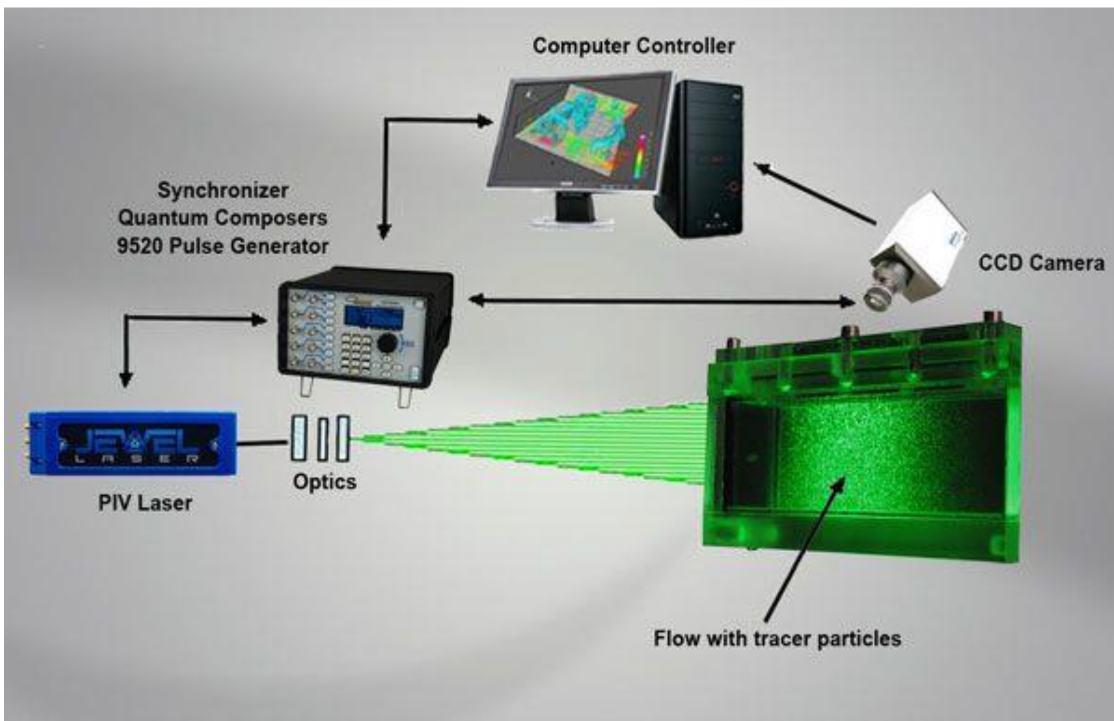
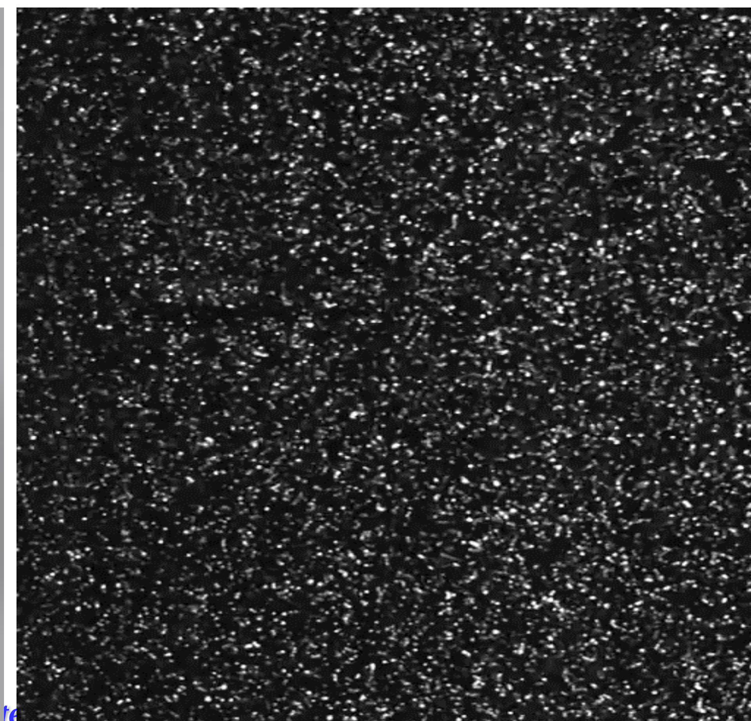
❑ PIV SYSTEM SETUP

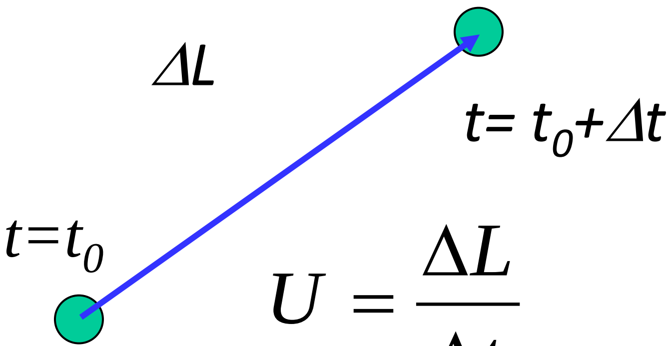
- ***Particle tracers:*** *to track the fluid movement.*
- ***Illumination system:*** *to illuminate the flow field in the interest region.*
- ***Camera:*** *to capture the images of the particle tracers.*
- ***Synchronizer:*** *to control the timing of the laser illumination & camera acquisition.*
- ***Host computer:*** *to store the particle images and conduct image processing.*



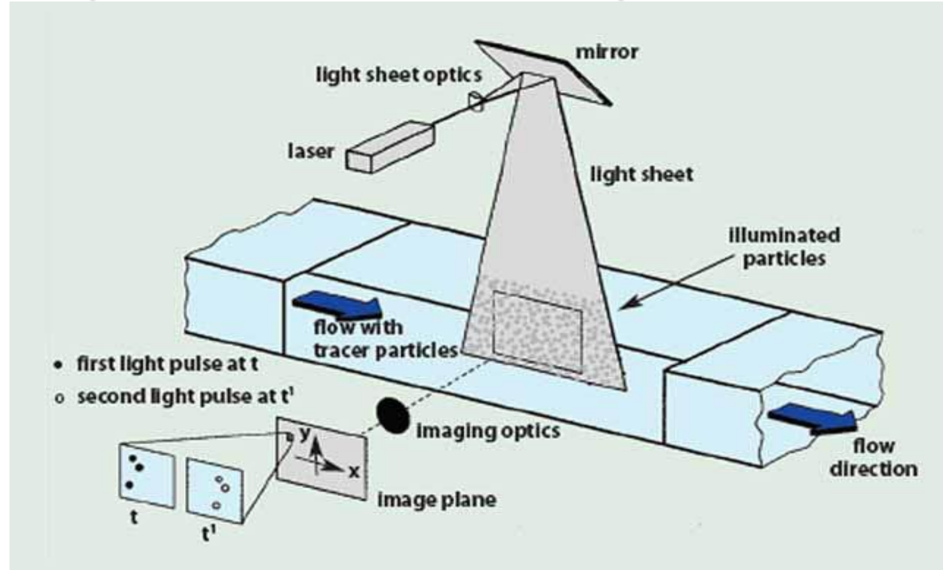


❑ PIV Image Processing

- ***Time-of-flight method:*** *to measure the displacements of the tracer particles seeded in the flow in a fixed time interval.*

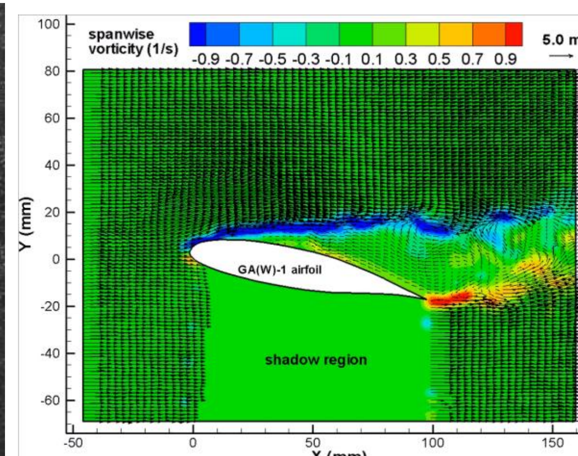


$$U = \frac{\Delta L}{\Delta t}$$



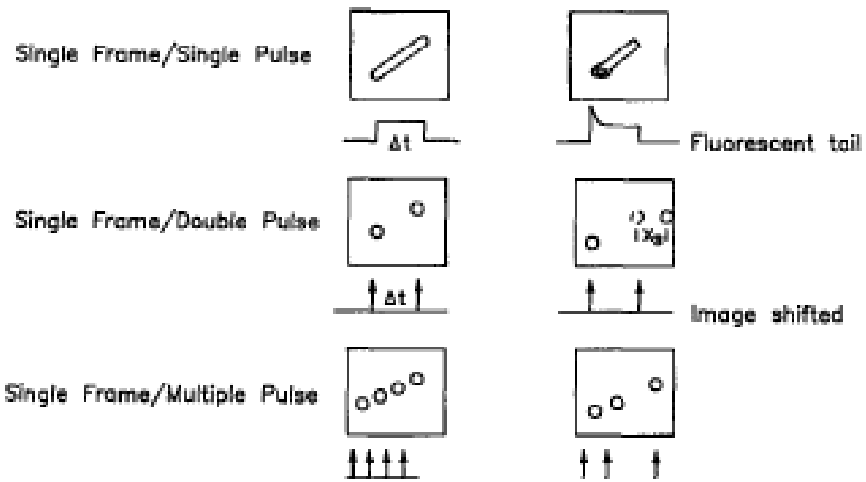
a. T=t0

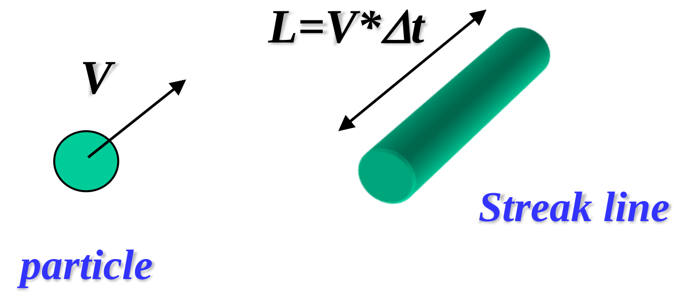
b. T=t0+10μs

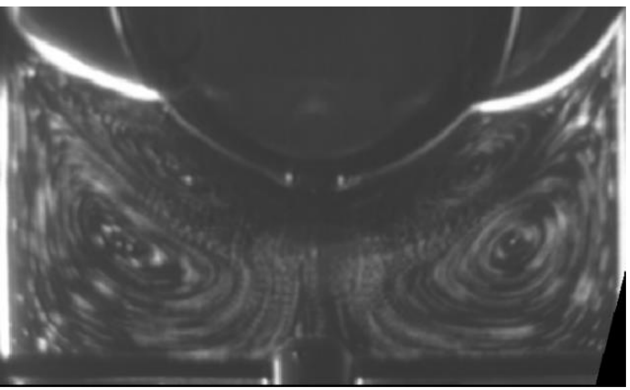


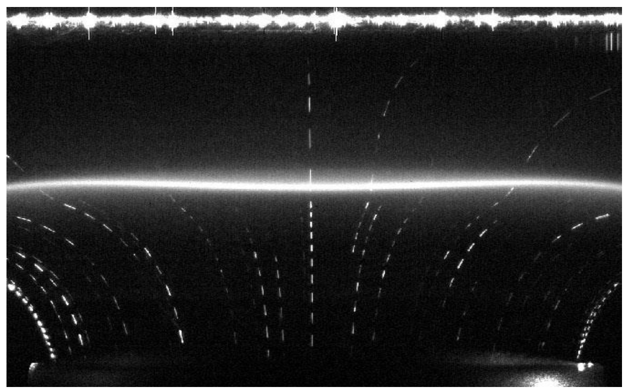
Corresponding Velocity field

❑ Single-frame PIV technique









single-pulse

Multiple-pulse

- *Particle streak velocimetry*
  - *Can measure the velocity magnitude.*
  - *Cannot determine the flow direction.*

Multi-frame technique

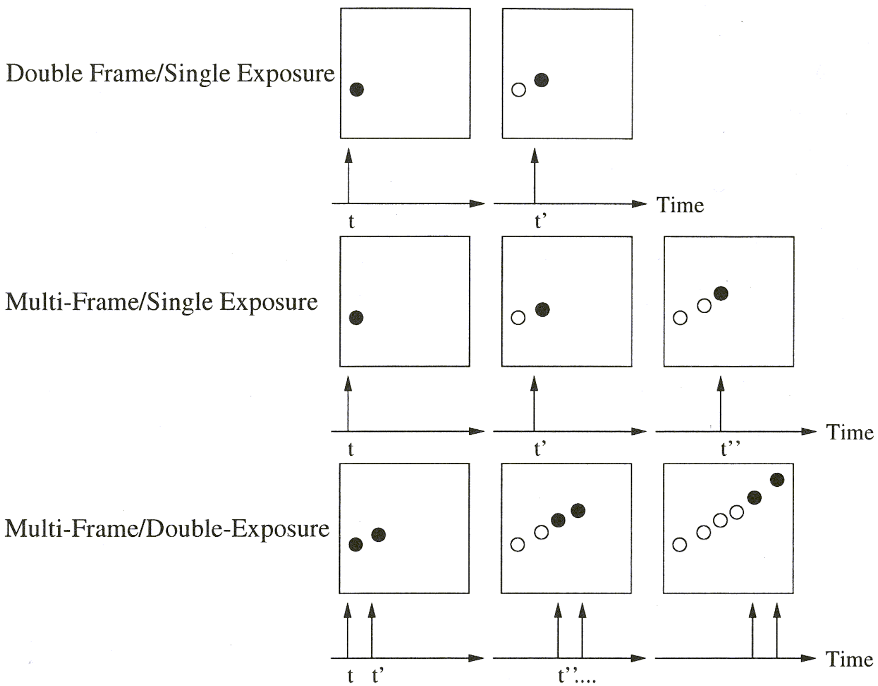
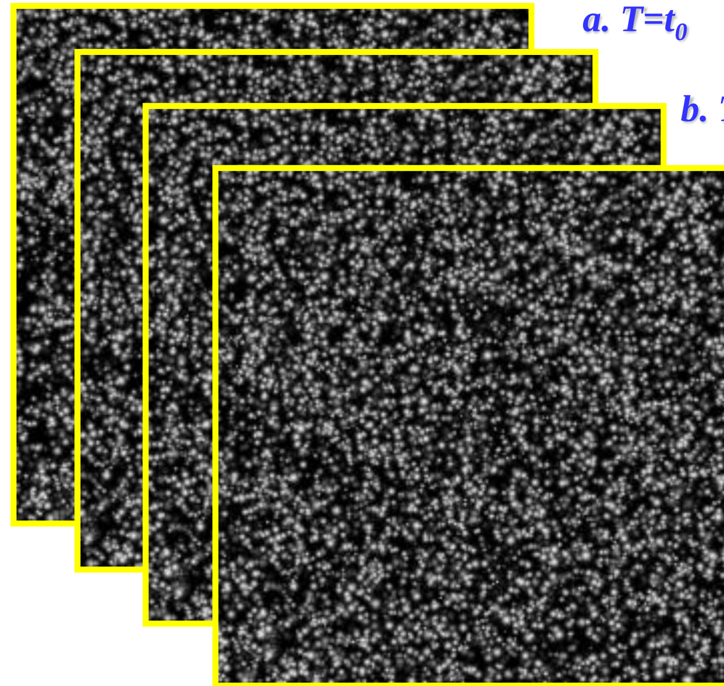
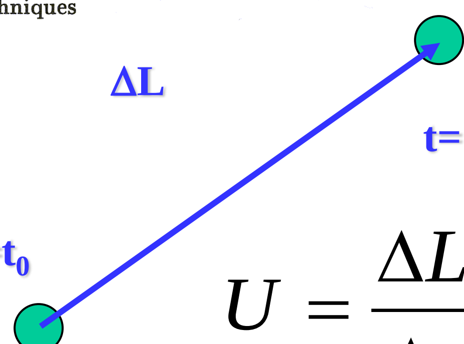
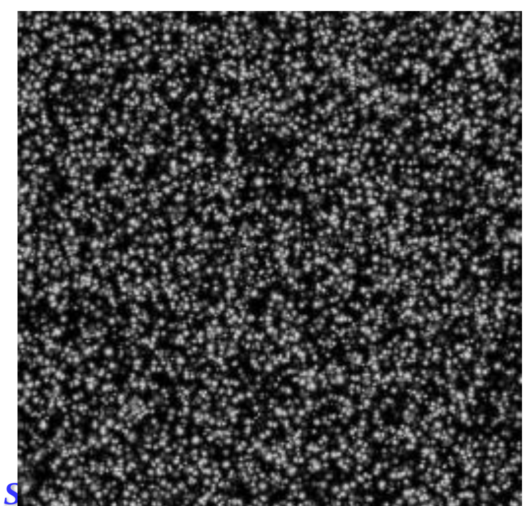


Fig. 4.2. Multiple frame techniques





$$U = \frac{\Delta L}{\Delta t}$$



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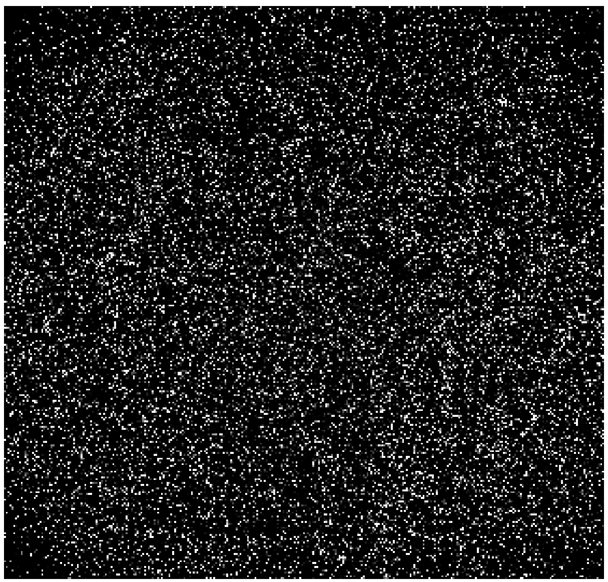
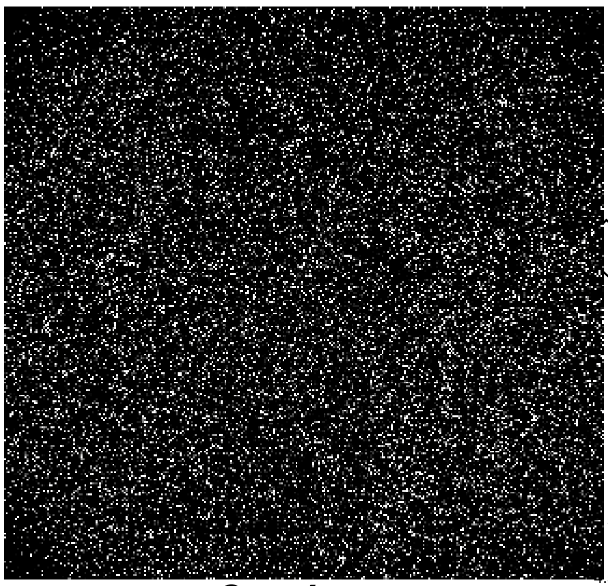
❑ Image Processing for PIV

- ***To extract velocity information from particle images.***

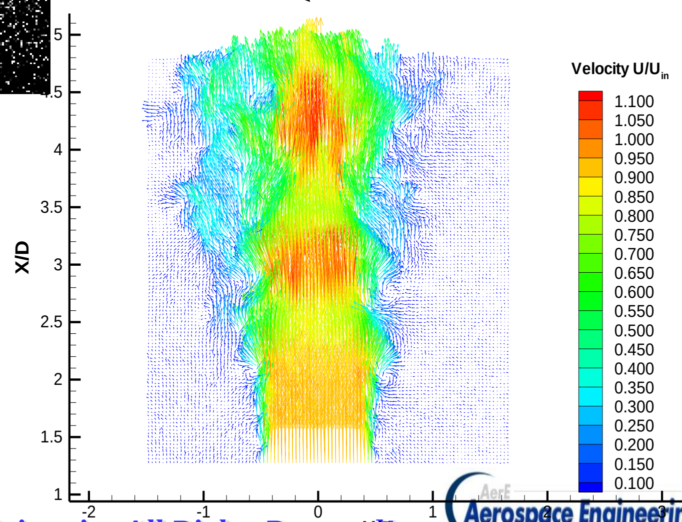


t=t0



t=t0+4ms

Image processing



A typical PIV raw image pair

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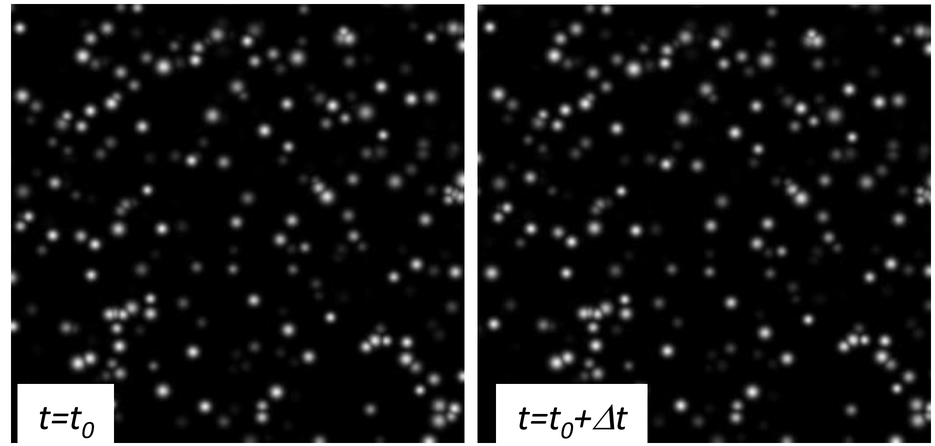
❑ Particle Tracking Velocimetry (PTV)

1. *Find position of the particles at each images*
2. *Find corresponding particle image pair in the different image frame*
3. *Find the displacements between the particle pairs.*
4. *Velocity of particle equates the displacement divided by the time interval between the frames.*



Low particle-image density case

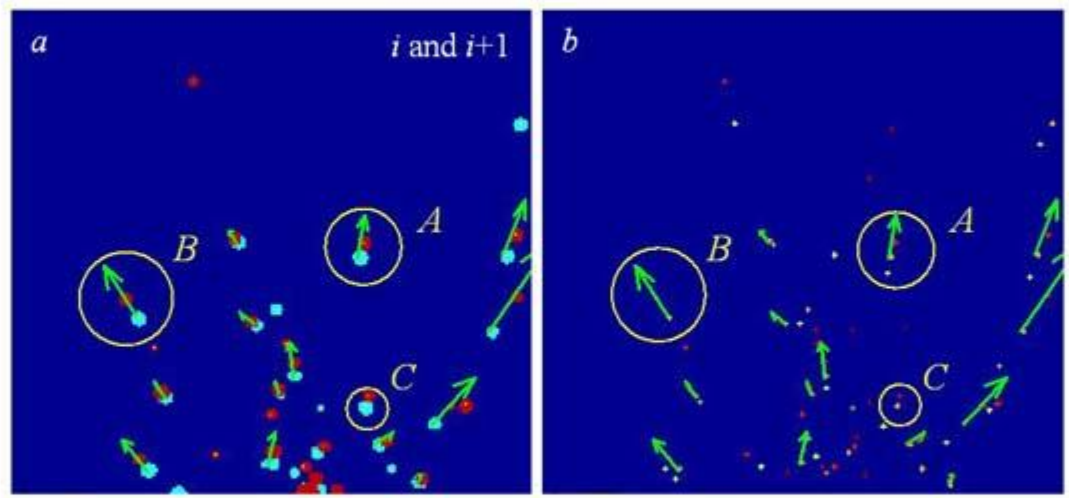


Figure 2. The particle-tracking algorithm applied to the sequence of images (i-1) to (i+2). (a) Detected particles in frames (i) (light blue) and (i+1) (dark red) with overlapped centroids and velocity vectors. (b) Detected centroids of particles in all four frames with overlaid velocity vectors. Consecutive frames are colored from light to dark.

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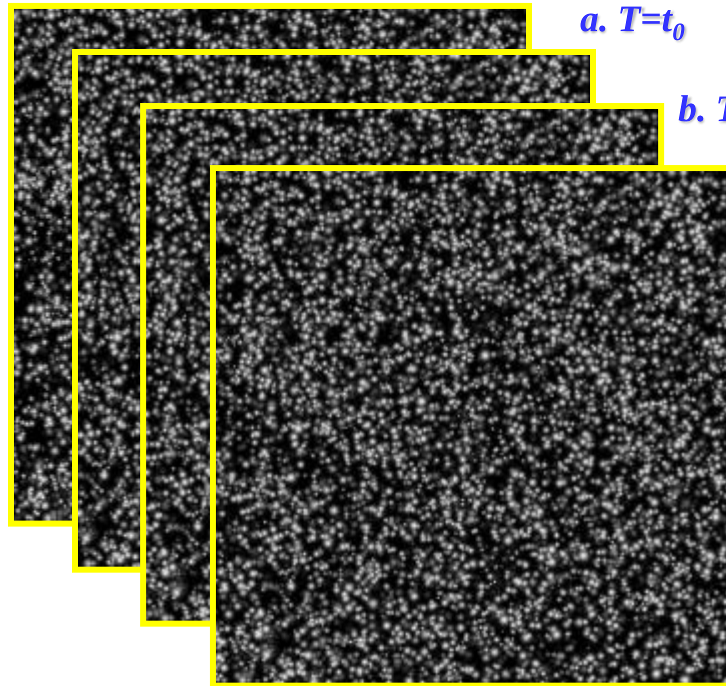
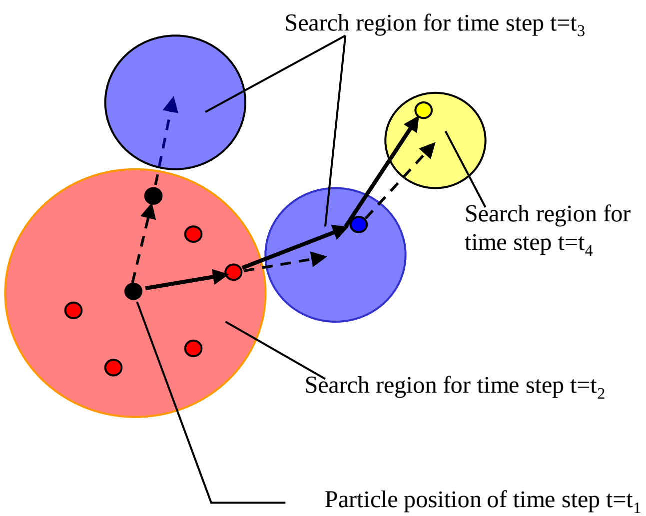
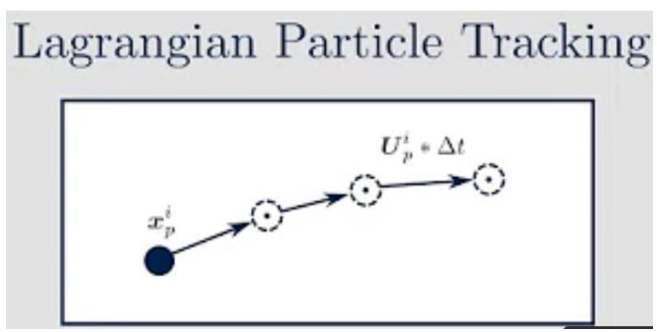
❑ Particle Tracking Velocimetry (PTV)-2

1. *Find position of the particles at each images*
2. *Find corresponding particle image pair in the different image frame*
3. *Find the displacements between the the particle pairs.*
4. *Velocity of particle equates the displacement divided by the time interval between the frames.*



results





- *Four-frame-particle tracking algorithm*

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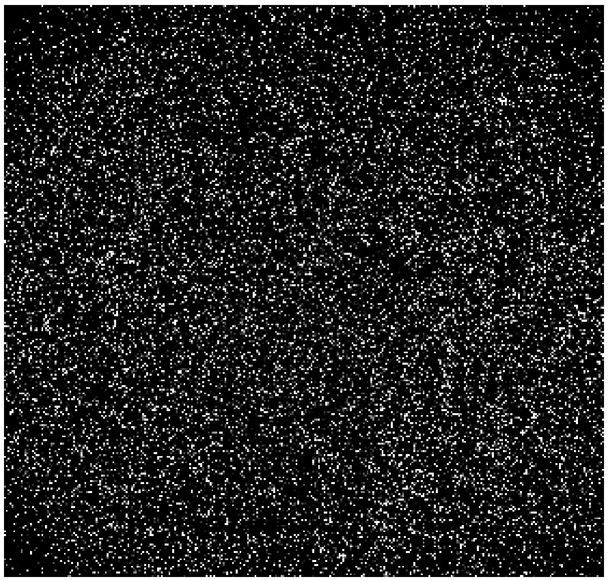
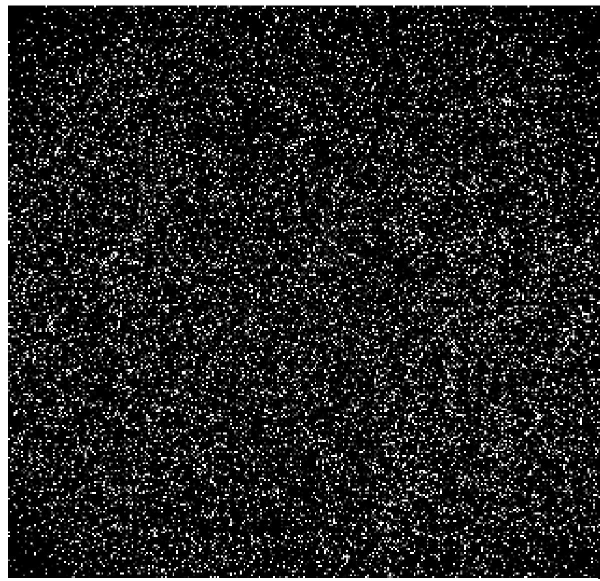
❑ IMAGE PROCESSING FOR PIV

- *To extract velocity information from particle images.*



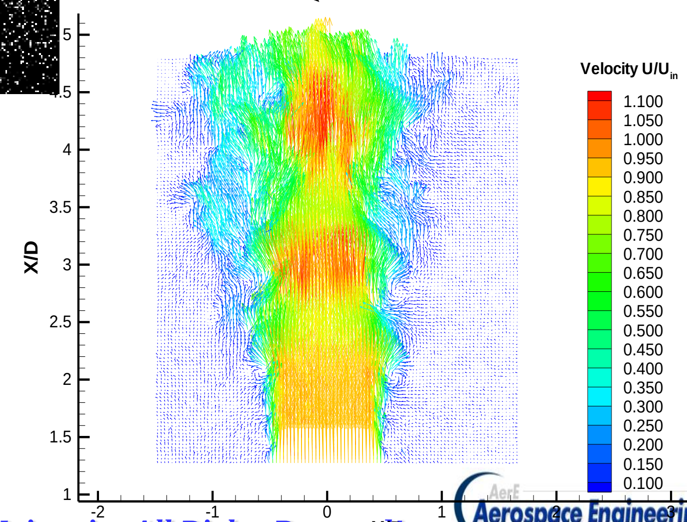
t=t0



t=t0+4ms

A typical PIV raw image pair

Image processing



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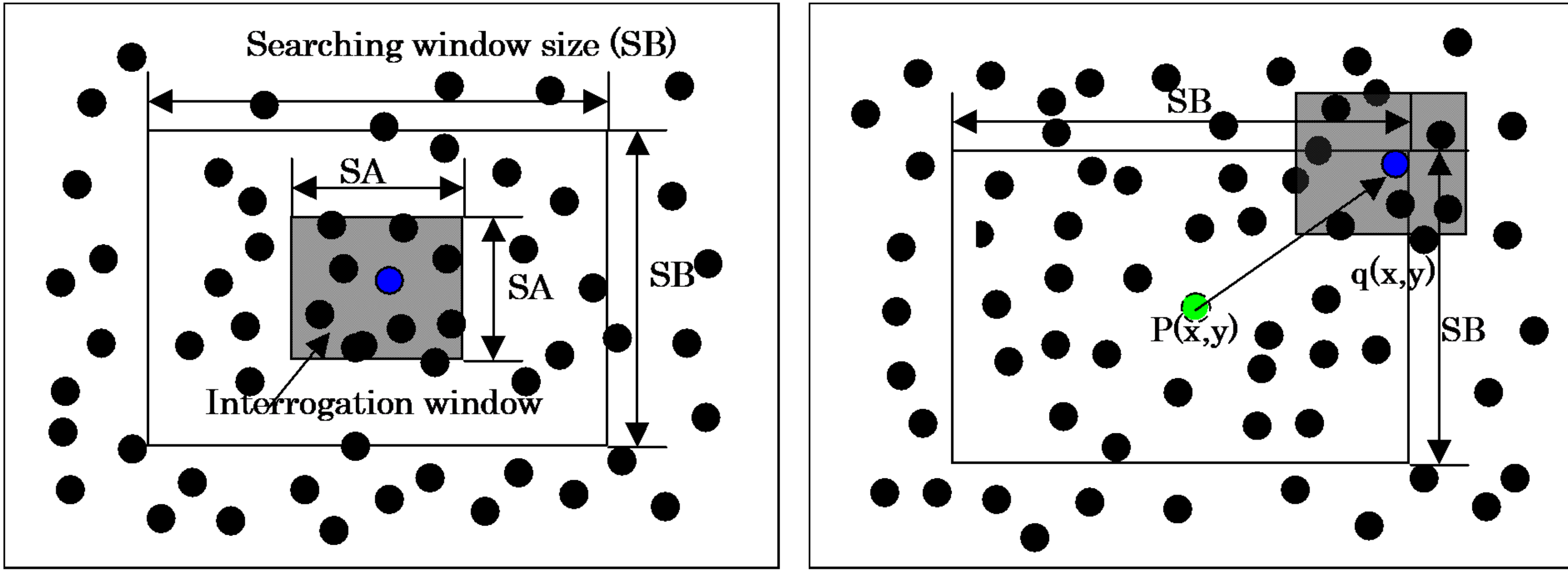
❑ Correlation-based PIV methods



$t=t_0$ $\qquad$ $t=t_0+\Delta t$

Correlation coefficient function

$$R(p,q)=\frac{\int (f(x,y)-\overline{f})(g(x,y)-\overline{g})dv}{\sqrt{\int (f(x,y)-\overline{f})^2 dv \int (g(x,y)-\overline{g})^2 dv}}$$

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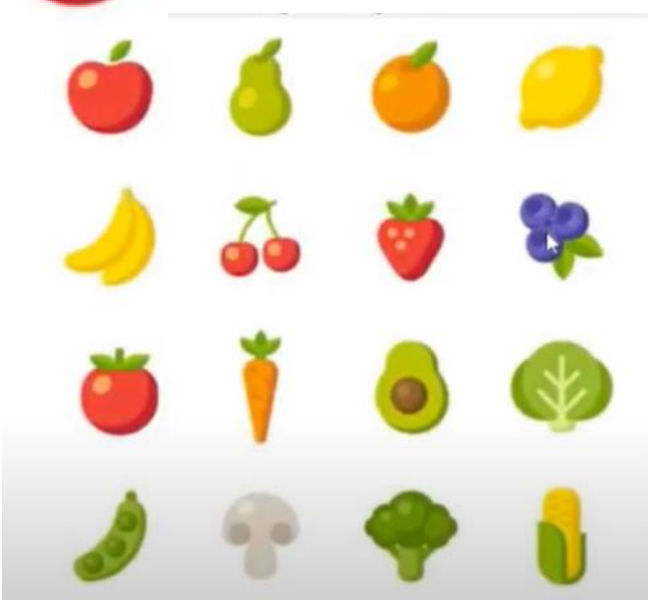
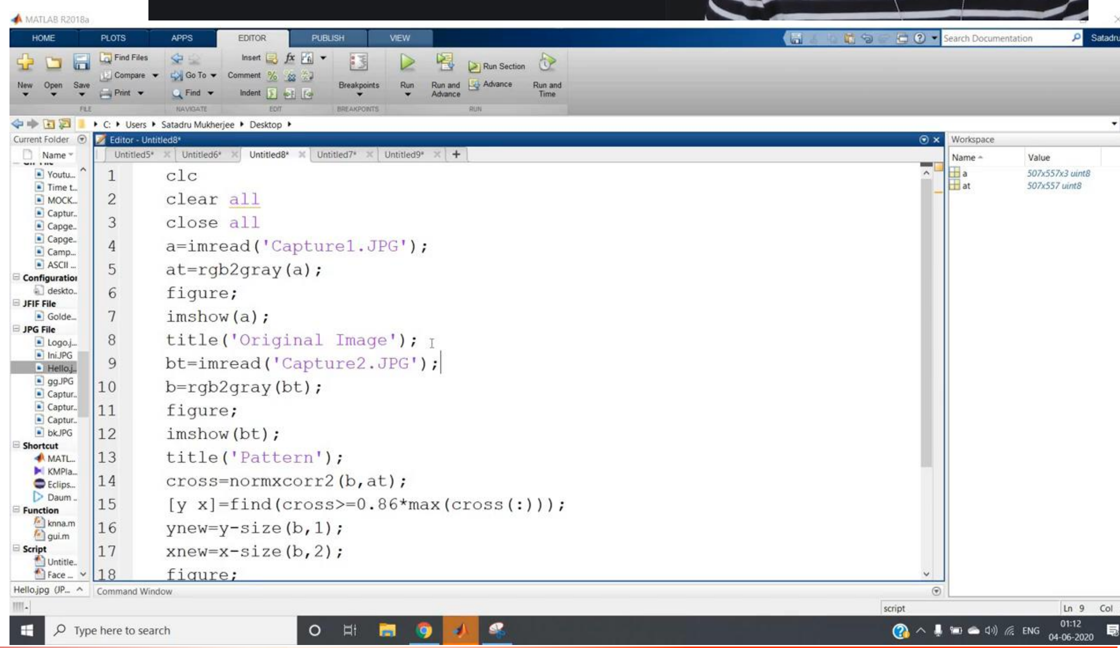
❑ Cross Correlation Operation

https://www.youtube.com/watch?v=cbZUnuyxcVs

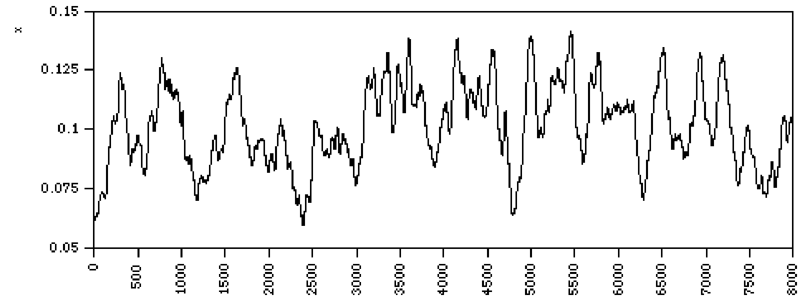
$$R(p,q)=\frac{\int(f(x,y)-\overline{f})(g(x,y)-\overline{g})dv}{\sqrt{\int(f(x,y)-\overline{f})^{2}dv\int(g(x,y)-\overline{g})^{2}dv}}$$



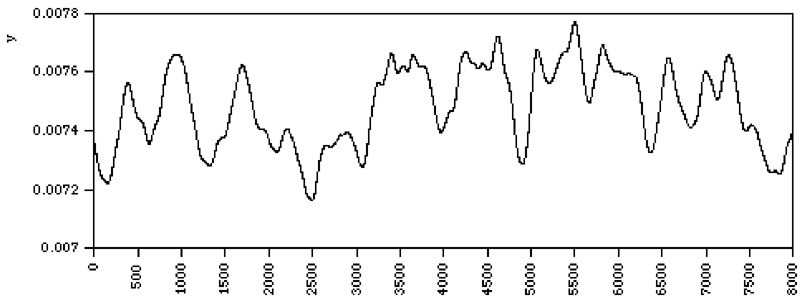


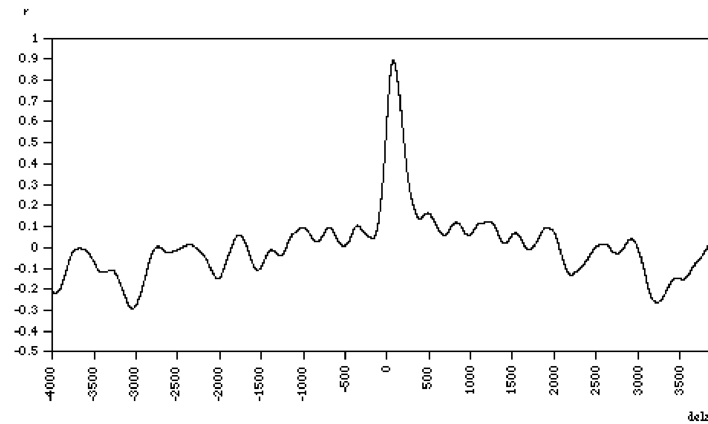
❑ Cross Correlation Operation

Signal A:

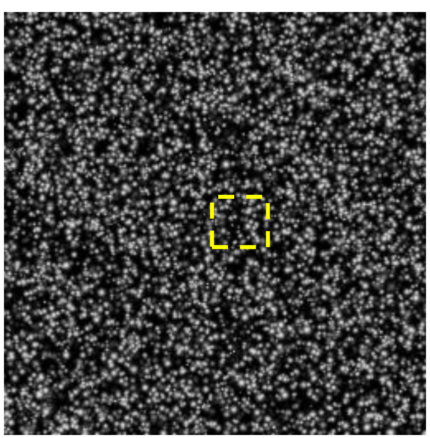


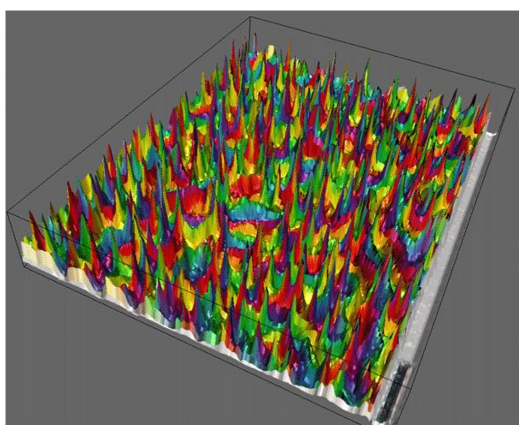
Signal B:

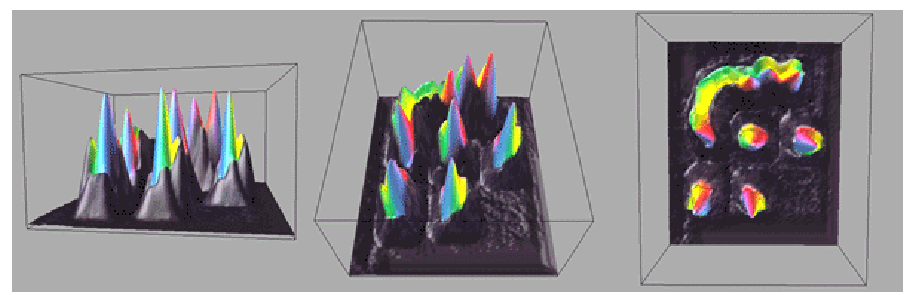




$$R(u) = \frac{\int [f(x) * g(x+u)]dx}{\sqrt{\int [f(x)^2]dx * \int [g(x+u)^2]dx}}$$







$$c(u,v) = \sum_{x,y} f(x,y)t(x-u, y-v)$$

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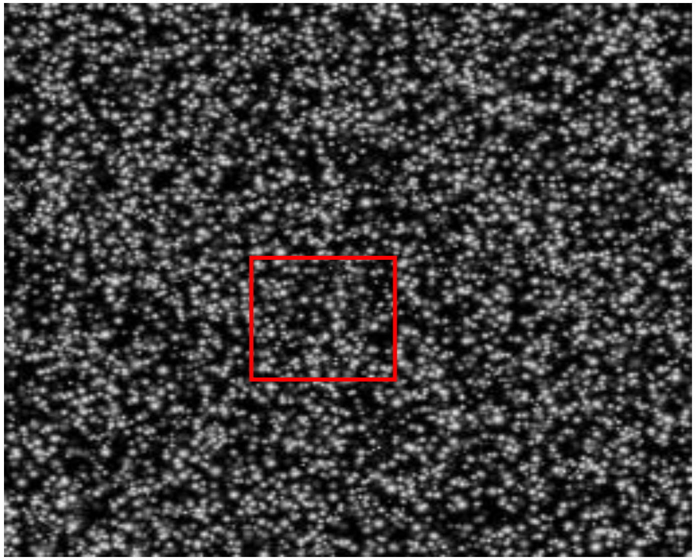
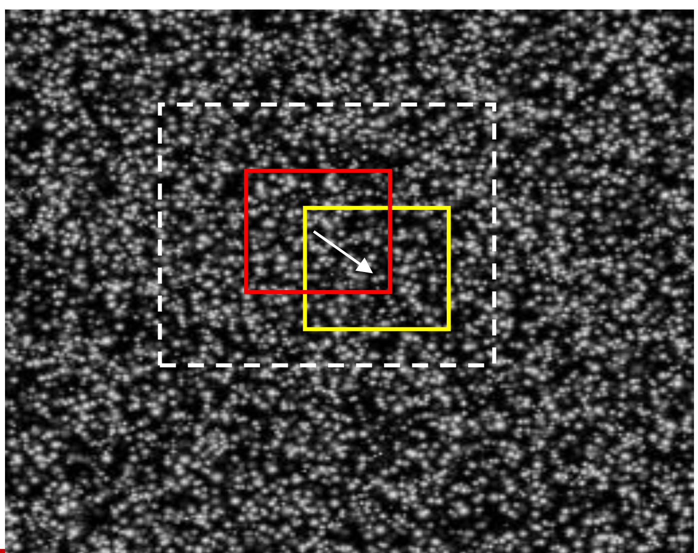
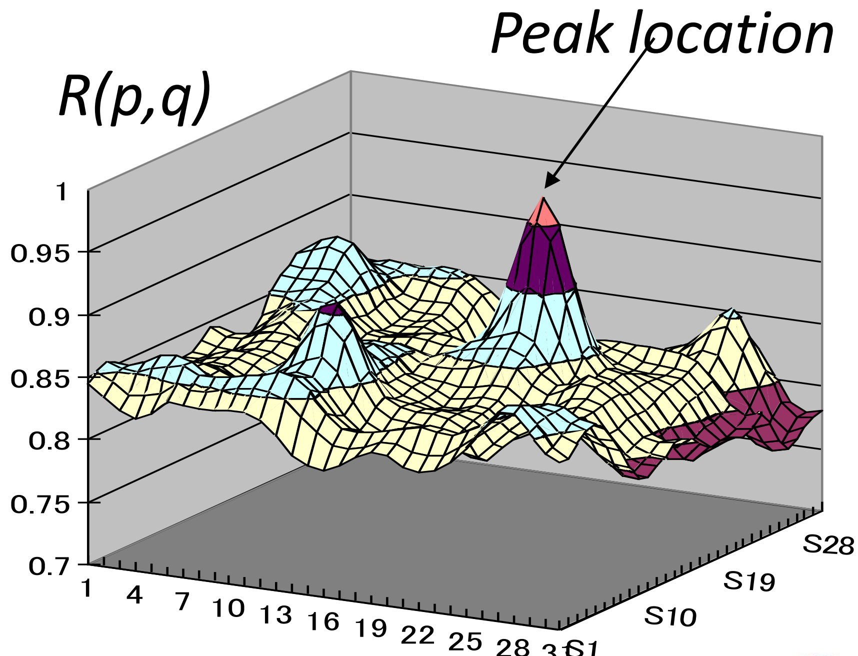
Correlation coefficient distribution





$$R(p,q)=\frac{\int (f(x,y)-\bar{f})(g(x,y)-\bar{g})dv}{\sqrt{\int (f(x,y)-\bar{f})^2 dv \int (g(x,y)-\bar{g})^2 dv}}$$



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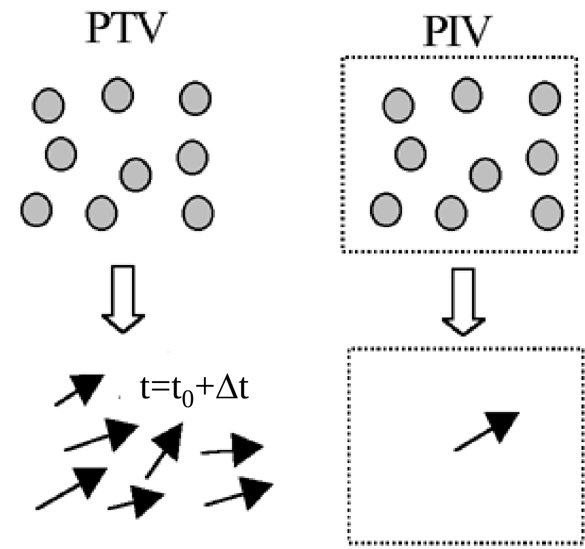
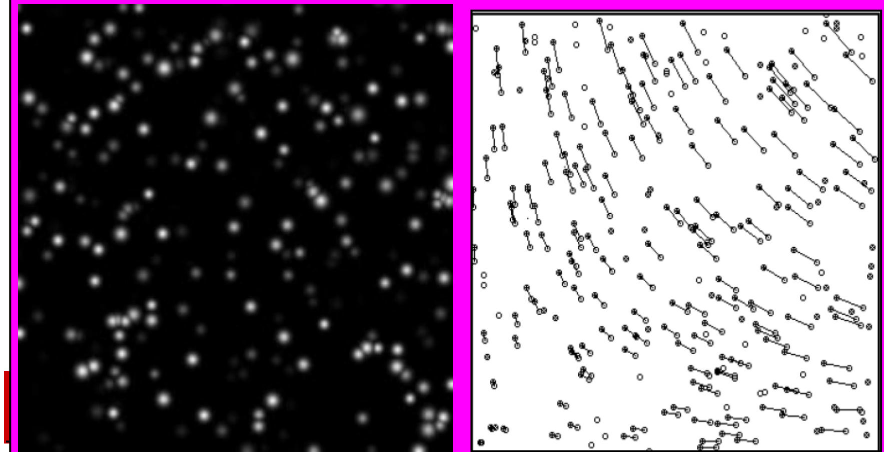
❑ Comparison between PIV and PTV

- *Particle Tracking Velocimetry:*
  - *Tracking individual particle*
  - *Limited to low particle image density case*
  - *Velocity vector at random points where tracer particles exist.*
  - *Spatial resolution of PTV results is usually limited by the number of the tracer particles*

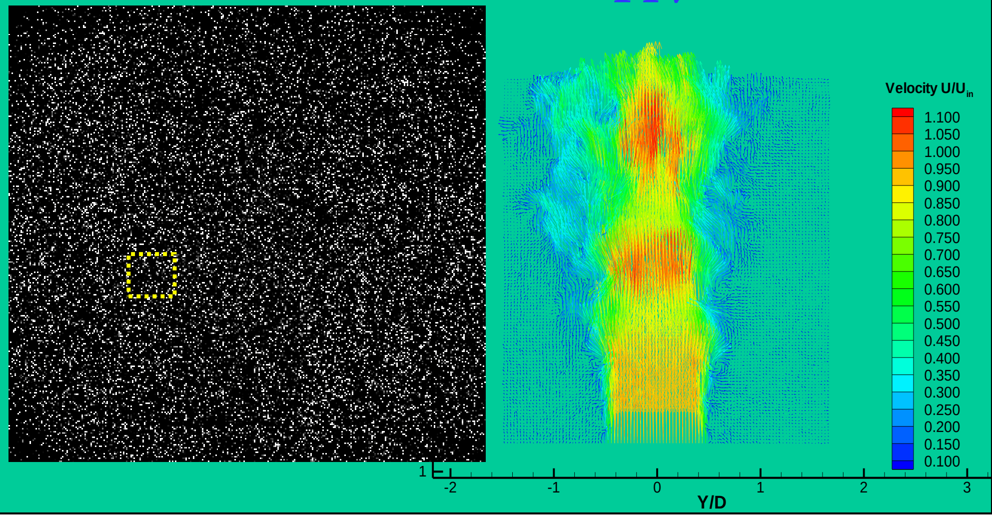
- *Correlation-based PIV:*
  - *Tracking a group of particles*
  - *Applicable to high particle image density case*
  - *Spatial resolution of PIV results is usually limited by the size of the interrogation window size*
  - *Velocity vector can be at regular grid points.*



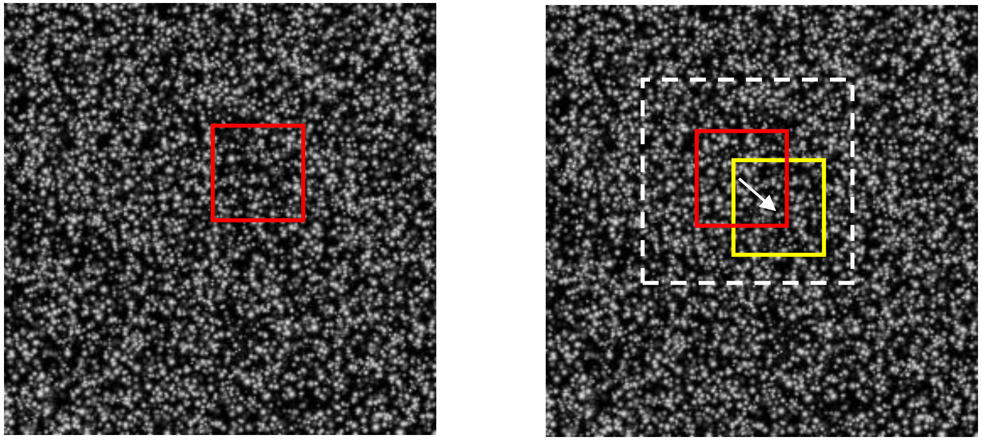
PTV



PIV



❑ FFT-based Cross Correlation and Direct Cross Correlation



$$R(p,q)=\frac{\int (f(x,y)-\overline{f})(g(x+p,y+q)-\overline{g})dv}{\sqrt{\int (f(x,y)-\overline{f})^2 dv \int (g(x+p,y+q)-\overline{g})^2 dv}}$$

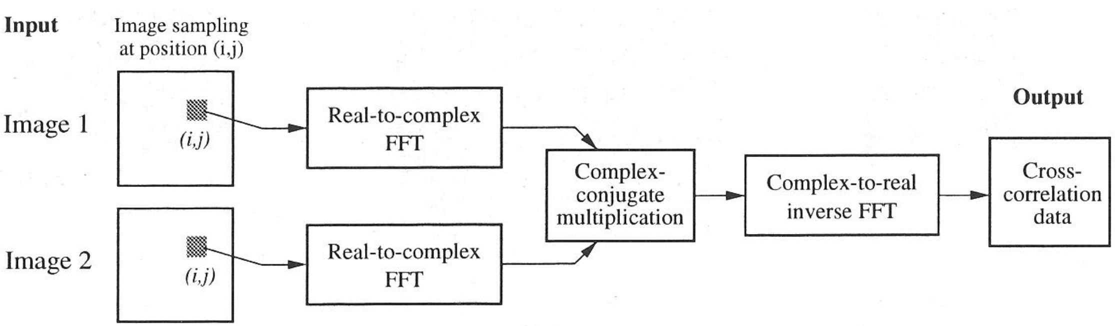
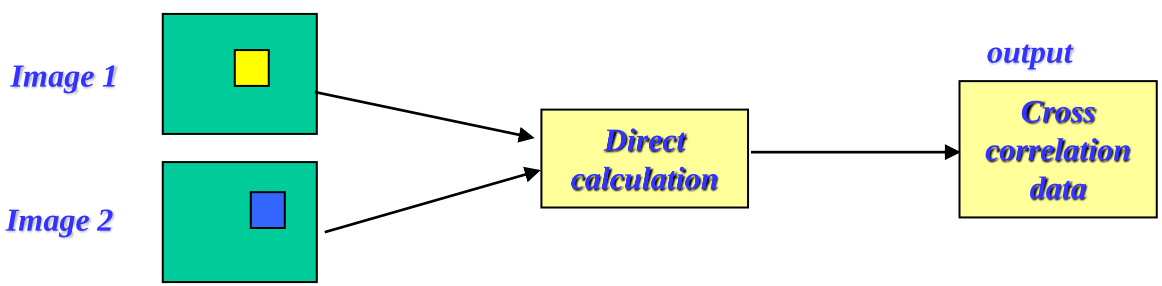


Fig. 5.16. Implementation of cross-correlation using fast Fourier transforms.

FFT-based Cross-correlation method

- ***advantage: Fast***
- ***disadvantage: additional error.***



Direct cross-correlation method

- ***advantage: accurate***
- ***disadvantage: time consuming***

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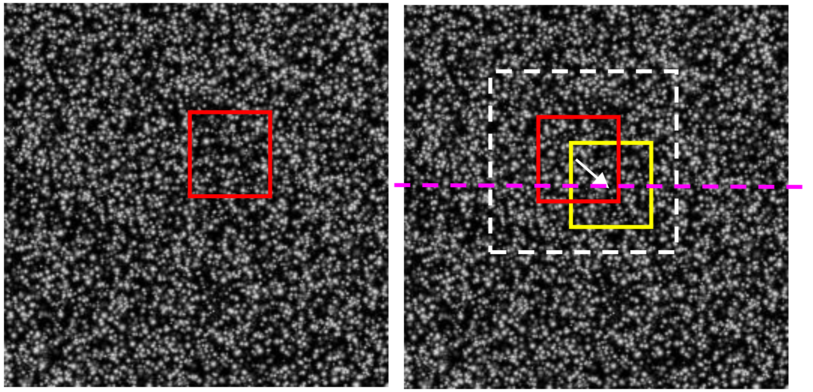
❑ Sub-pixel interpolation for Digital PIV

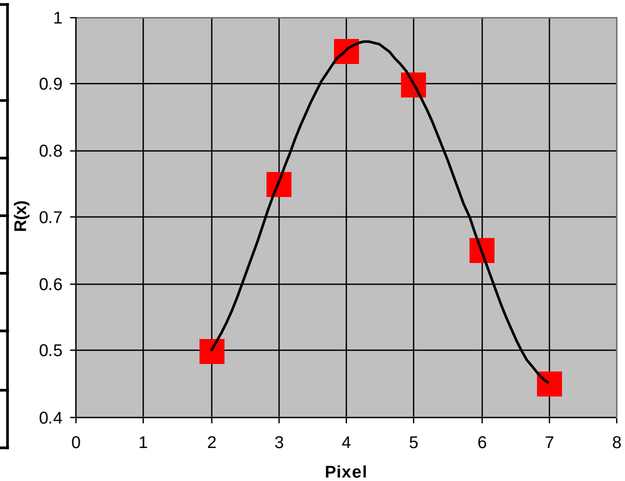


$$R(p,q)=\frac{\int (f(x,y)-\overline{f})(g(x+p,y+q)-\overline{g})dv}{\sqrt{\int (f(x,y)-\overline{f})^2 dv \int (g(x+p,y+q)-\overline{g})^2 dv}}$$

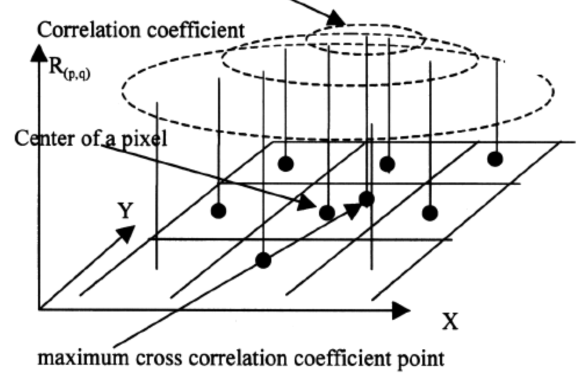
Table 5.1. Three-point estimators for determining the displacement from the correlation data at the subpixel level

Fitting function	Estimators
Peak centroid $f(x)=\frac{\text{first order moment}}{\text{zero order moment}}$	$x_0=\frac{(i-1)R_{(i-1,j)}+iR_{(i,j)}+(i+1)R_{(i+1,j)}}{R_{(i-1,j)}+R_{(i,j)}+R_{(i+1,j)}}$ $y_0=\frac{(j-1)R_{(i,j-1)}+jR_{(i,j)}+(j+1)R_{(i,j+1)}}{R_{(i,j-1)}+R_{(i,j)}+R_{(i,j+1)}}$
Parabolic peak fit $f(x)=Ax^2+Bx+C$	$x_0=i+\frac{R_{(i-1,j)}-R_{(i+1,j)}}{2R_{(i-1,j)}-4R_{(i,j)}+2R_{(i+1,j)}}$ $y_0=j+\frac{R_{(i,j-1)}-R_{(i,j+1)}}{2R_{(i,j-1)}-4R_{(i,j)}+2R_{(i,j+1)}}$
ussian peak fit $f(x)=C\exp\left[\frac{-(x_0-x)^2}{k}\right]$	$x_0=i+\frac{\ln R_{(i-1,j)}-\ln R_{(i+1,j)}}{2\ln R_{(i-1,j)}-4\ln R_{(i,j)}+2\ln R_{(i+1,j)}}$ $y_0=j+\frac{\ln R_{(i,j-1)}-\ln R_{(i,j+1)}}{2\ln R_{(i,j-1)}-4\ln R_{(i,j)}+2\ln R_{(i,j+1)}}$

pixel	Correlation coefficient
2	0.50
3	0.75
4	0.95
5	0.90
6	0.65
7	0.45



$$\begin{cases} R_{(p,q)}=a\exp(-b(x-c)).\exp(-d(y-e)) & (1) \\ R_{(p,q)}=ax^2+by^2+cxy+dx+ey+f & (2) \end{cases}$$



- *With sub-pixel interpolation processing, the accuracy of the PIV measurement could be about 0.1 pixel*

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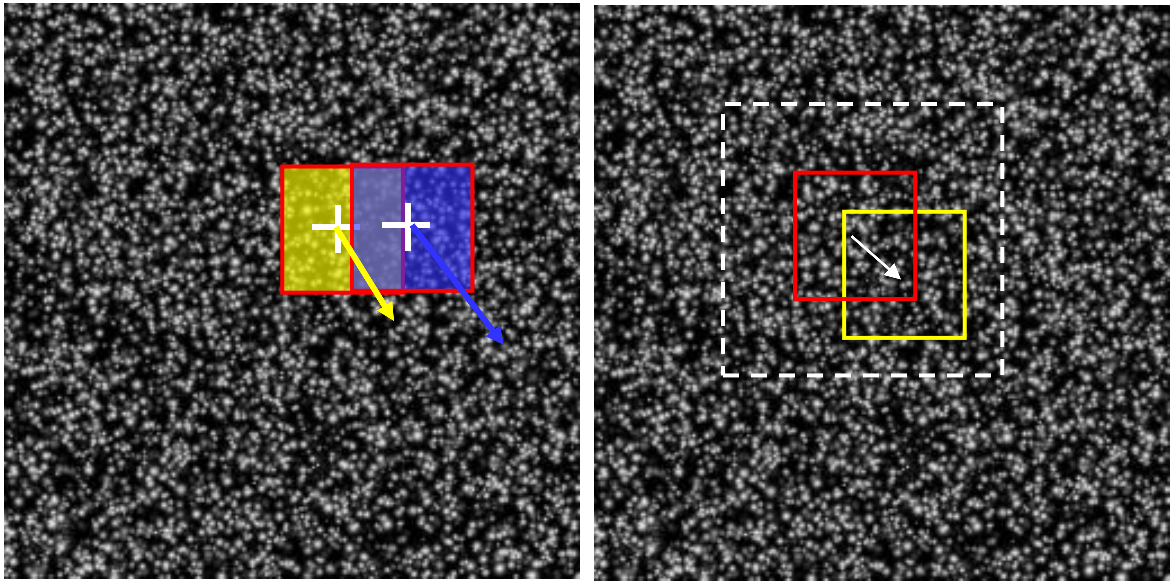
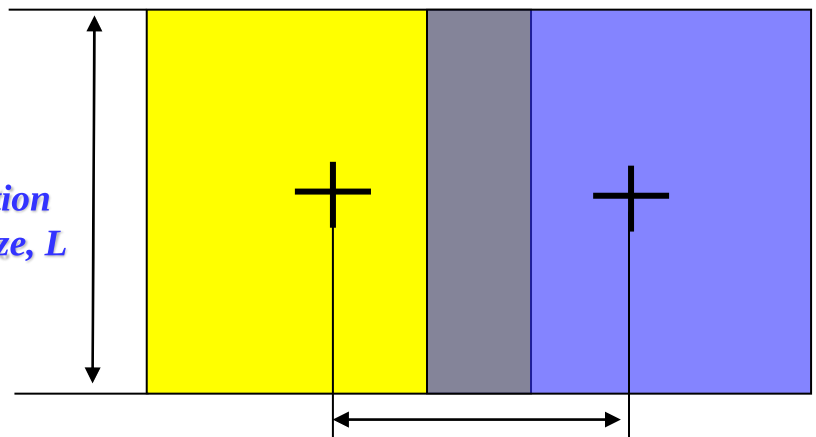
Overlapping rate for PIV image processing



$$R(p,q)=\frac{\int (f(x,y)-\overline{f})(g(x+p,y+q)-\overline{g})dv}{\sqrt{\int (f(x,y)-\overline{f})^2 dv \int (g(x+p,y+q)-\overline{g})^2 dv}}$$



Overlapping rate = 1.0 - D / S

50% overlapping is usually used for PIV image processing!!

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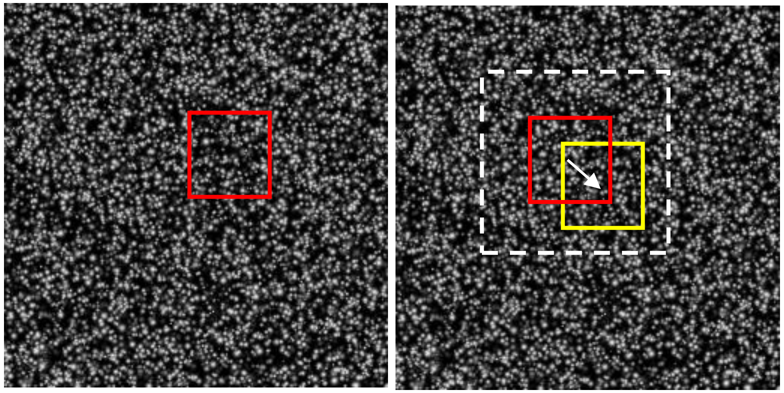
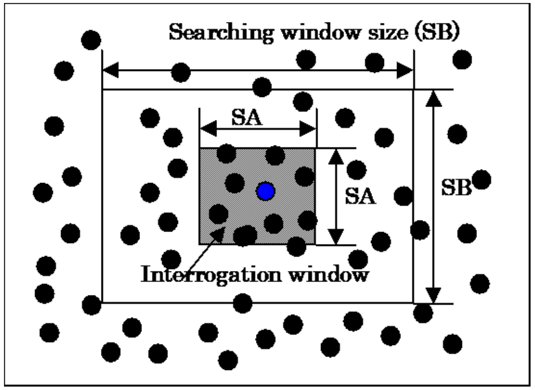
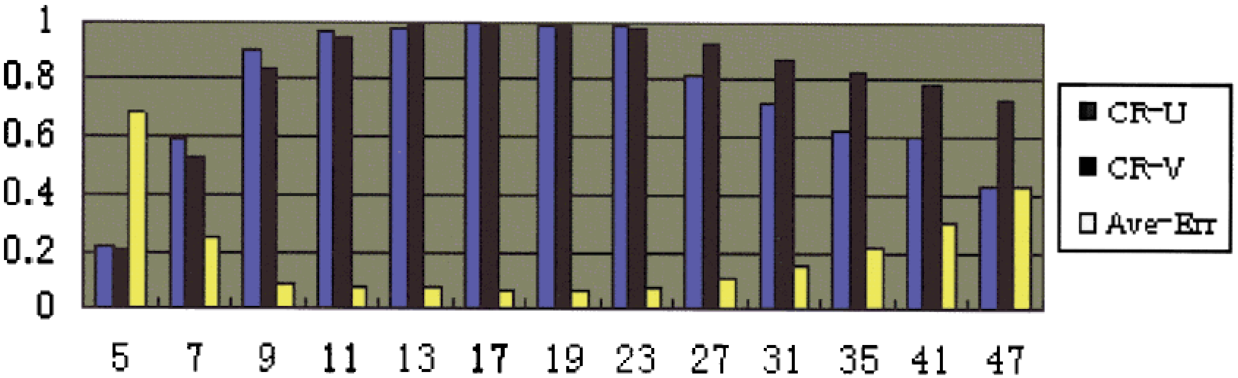
❑ EFFECT OF INTERROGATION WINDOW SIZE

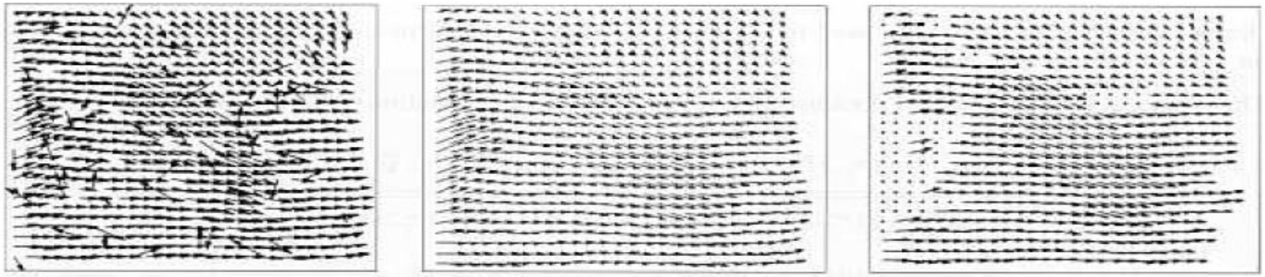
- *Interrogation size usually determines the spatial resolution of the PIV measurements.*
- *Smaller interrogation window size will give a better spatial resolution of the PIV measurement.*
- *However, too small interrogation window size would result in many bad vectors.*
- *Usually, to have about 10 ~ 20 particles inside an interrogation window would give a good PIV result!*



$$R(p,q)=\frac{\int (f(x,y)-\overline{f})(g(x+p,y+q)-\overline{g})dv}{\sqrt{\int (f(x,y)-\overline{f})^2 dv \int (g(x+p,y+q)-\overline{g})^2 dv}}$$







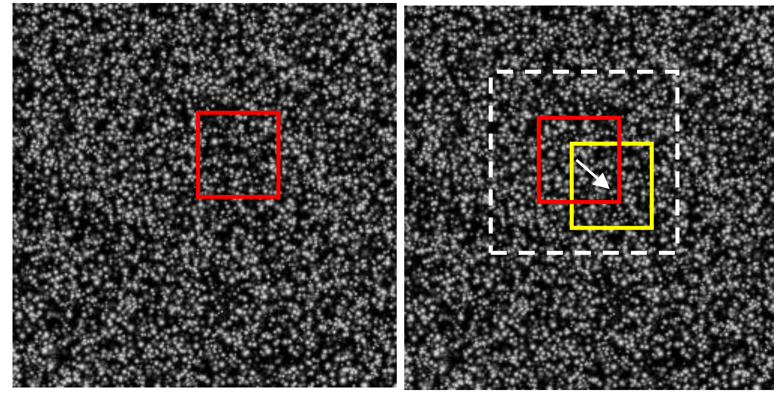
A. SA=7, SB=29 B. SA=17, SB=29 C. SA=35, SB=29

- *<u>Hu H</u>, **Saga T, Kobayashi T, Okamoto T, Taniguchi N**, "Evaluation of Cross-Correlation Method by Using PIV Standard Images", Journal of Visualization, Vol.1, No.1, pp87-94, 1998*

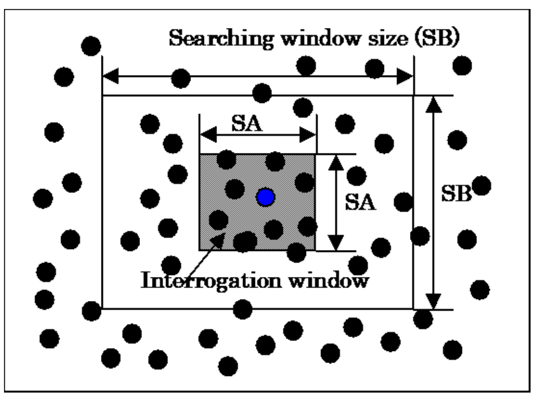
❑ Effect of Search Window Size

- *The size of the search window size would determine total time required for the cross correlation processing*

- *Smaller search window size could save the computational time, however, would result in error vectors for the particles with larger displacement.*



$$R(p,q)=\frac{\int(f(x,y)-\overline{f})(g(x+p,y+q)-\overline{g})dv}{\sqrt{\int(f(x,y)-\overline{f})^{2}dv\int(g(x+p,y+q)-\overline{g})^{2}dv}}$$



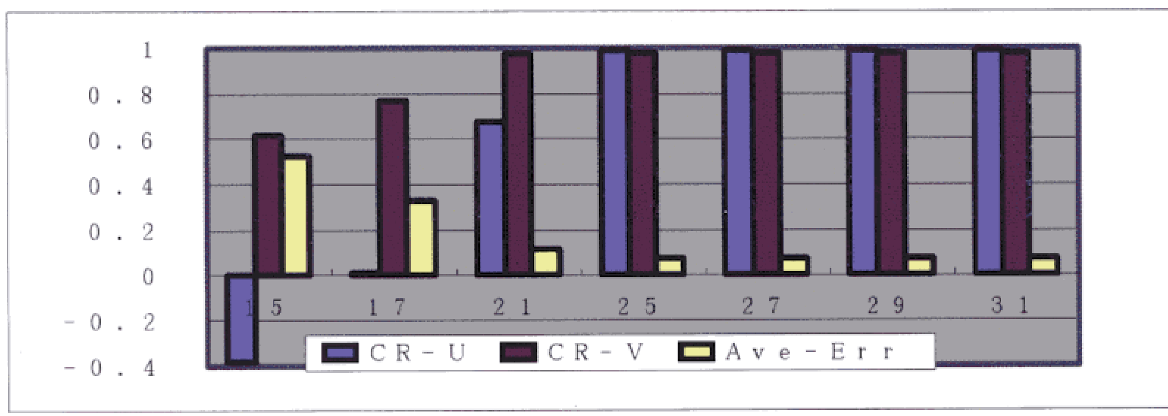


Fig. 5. The effect of the search window size (case A, SA=17).

- **<u>Hu H</u>, Saga T, Kobayashi T, Okamoto T, Taniguchi N**, *"Evaluation of Cross-Correlation Method by Using PIV Standard Images", Journal of Visualization, Vol.1, No.1, pp87-94, 1998*

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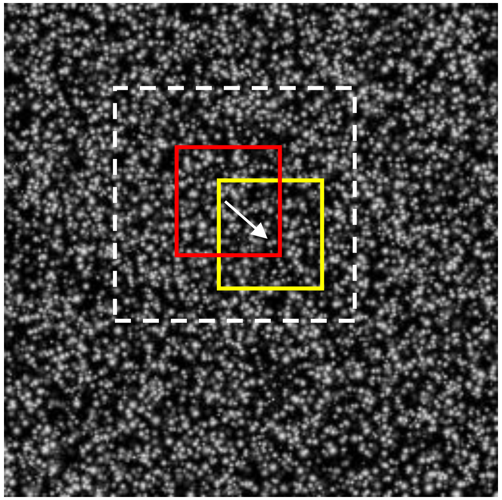
❑ Effect of the Displacements of the Tracer Particles or the Time Delay between the Two Laser Pulses

- *Too large displacement of the particles (i.e., longer time delay) would result in bigger errors due to the relative movement of the particles inside the interrogation window.*
- *Too small displacement of the particles (i.e., smaller time delay) would also result in bigger errors due to the relative small displacement related to the limited resolution of the digital camera.*
- *It usually to have particle displacement about 4~6 pixels if the interrogation size is chosen to be 32 pixels.*



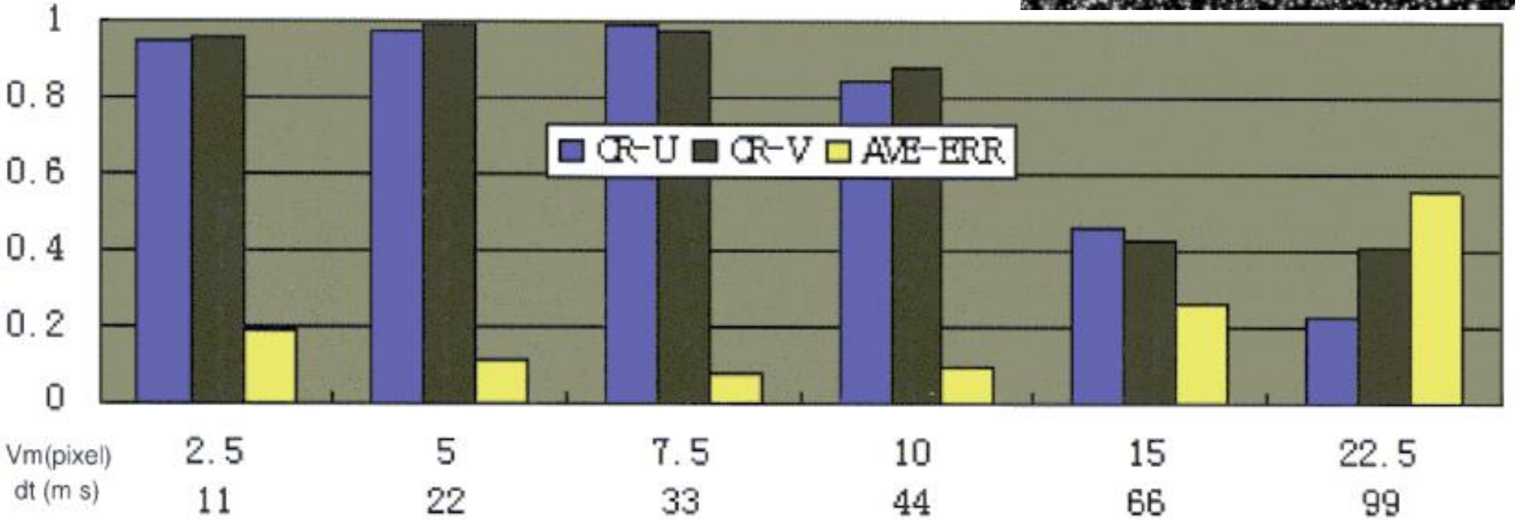


Fig. 6. The effect of the average image velocity or time interval of the two images.

- ***Hu H**, Saga T, Kobayashi T, Okamoto T, Taniguchi N*, *"Evaluation of Cross-Correlation Method by Using PIV Standard Images", Journal of Visualization, Vol.1, No.1, pp87-94, 1998*

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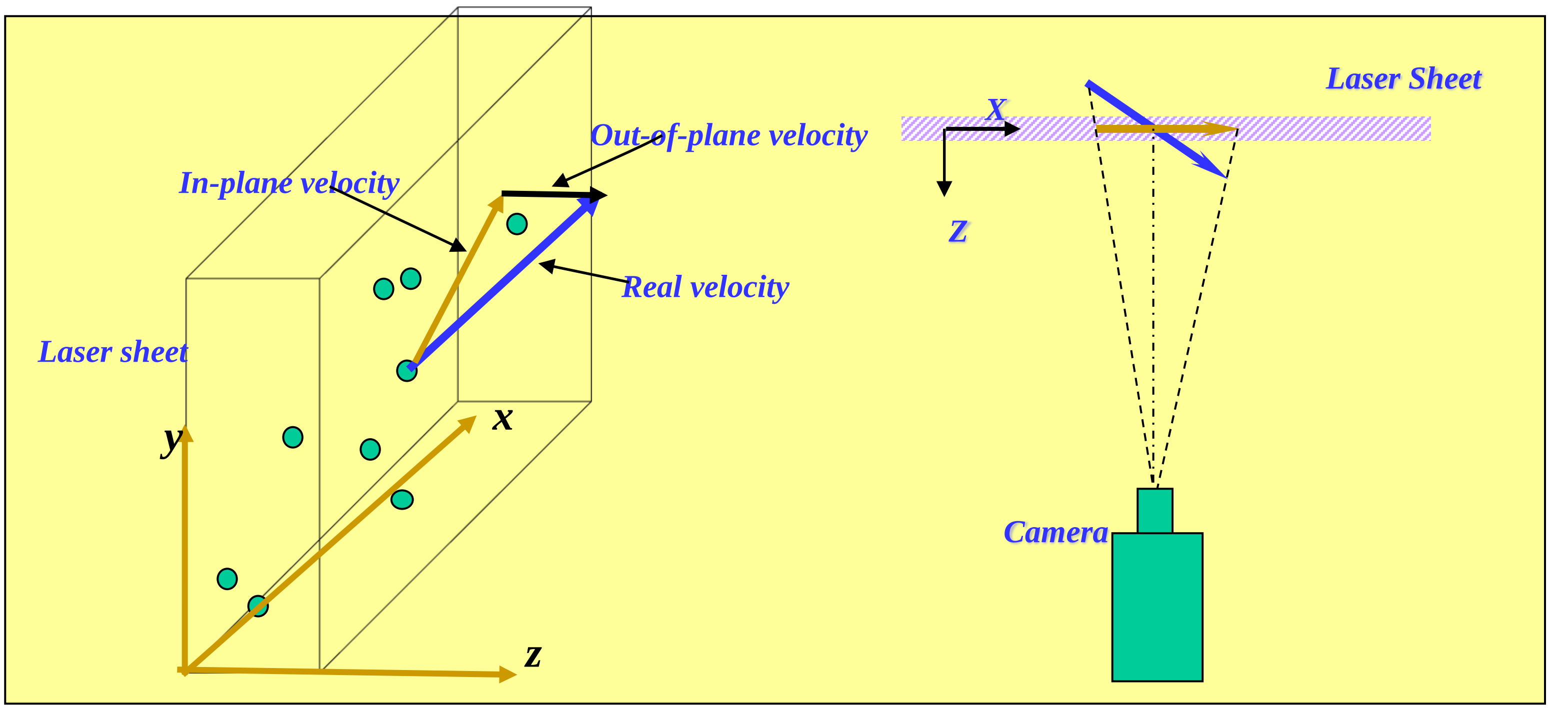
❑ Effect of the out-of-plan velocity for 2-D PIV measurements

case	V_M	V_X	W_M	N	D_M	CR-U	CR-V	AVE-ERR
A	7.5	15.0	0.017	4,000	5.0	0.991	0.980	7.12%
F	7.5	15.0	0.17	4,000	5.0	0.698	0.748	13.75%
G	7.5	15.0	0.34	4,000	5.0	0.363	0.394	41.30%
H	7.5	15.0	1.70	4,000	5.0	-0.06	-0.05	132.1%

V_M : average velocity (pixel/interval)
V_X : maximum Velocity (pixel/interval)
W_M : out of plane velocity (laser width/interval)
N: tracer number
D_M : tracer average diameter (pixel)
Aver-Err: average error of PIV results without sub-pixel interpolation.



- ***<u>Hu H</u>, Saga T, Kobayashi T, Okamoto T, Taniguchi N***, *"Evaluation of Cross-Correlation Method by Using PIV Standard Images", Journal of Visualization, Vol.1, No.1, pp87-94, 1998*

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❑ Effect of the Diameter of Particle Images

CASE	V_M	V_X	W_M	N	D_M	CR-U	CR-V	AVE-ERR
1	7.5	15.0	0.017	4,000	2.5	0.916	0.871	9.05%
2	7.5	15.0	0.017	4,000	5.0	0.991	0.980	7.12%
3	7.5	15.0	0.017	4,000	10.0	0.992	0.982	6.98%

V_M: average velocity (pixel/interval)
V_X: maximum Velocity (pixel/interval)
W_M: out of plane velocity (laser width/interval)
N: tracer particle number
D_M: tracer average diameter (pixel)
Aver-Err: average error of PIV results without sub-pixel interpolation.

- *Bigger particles maybe beneficial for PIV image processing*
- *Bigger particles would cause the problem for the unsteady flow tracking !*

- ***Hu H, Saga T, Kobayashi T, Okamoto T, Taniguchi N***, *"Evaluation of Cross-Correlation Method by Using PIV Standard Images", Journal of Visualization, Vol.1, No.1, pp87-94, 1998*

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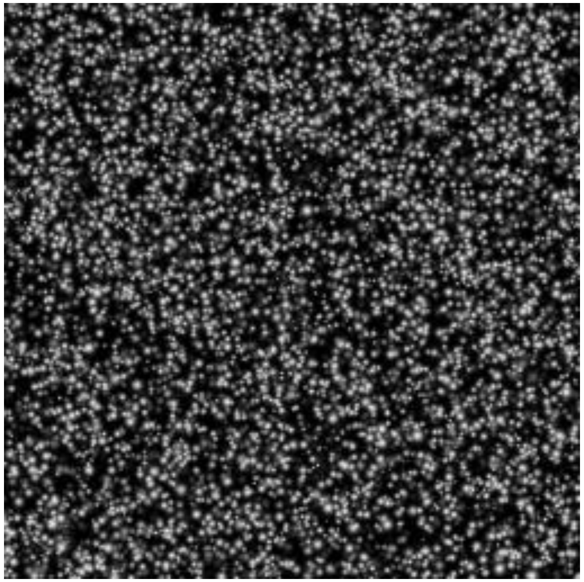
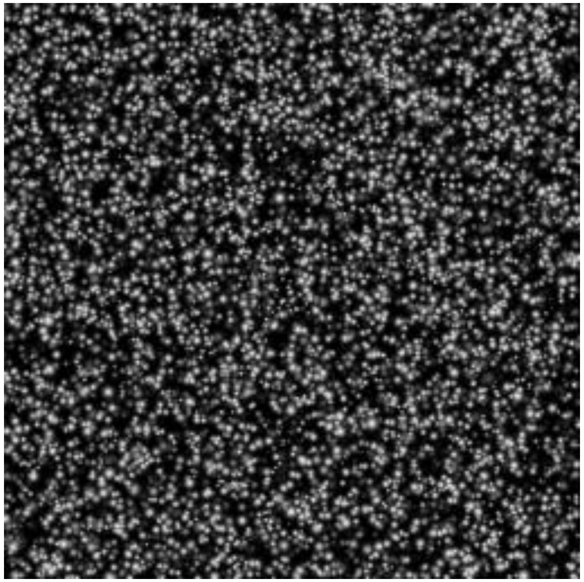
❑ HOMEWORK ASSIGNMENT

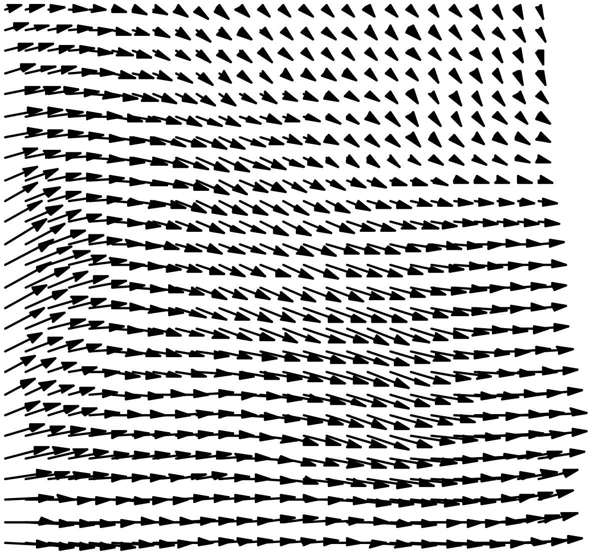
- *Develop your own PIV image processing code using correlation-based method.*
- *Standard PIV images are available at http://www.vsj.jp/~pivstd/image-e.html*
- *You can use C++, Matlab, Labview or other software you are familiar with, but not commercial PIV software, please.*
- *You can use the code you developed for the PIV image processing of the Lab #4.*
- *You can work as a group (2~4 persons)or individually.*



$t=t_0$



$t=t_0+\Delta t$



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